

7.4: 14, 16, 24, 28, 32, 42.

14)

$$a) \int \frac{y}{y+2} \cdot dy$$

$$= \int \left(1 - \frac{2}{y+2}\right) dy$$

$$= y - 2 \ln |y+2| + C.$$

16)

$$b) \int \frac{1}{(t+4)(t-1)} dt$$

$$= \int \left(\frac{-1/5}{t+4} + \frac{1/5}{t-1}\right) dt$$

$$= -1/5 \cdot \ln |t+4| + 1/5 \cdot \ln |t-1| + C.$$

$$= \frac{1}{5} \ln \left| \frac{t-1}{t+4} \right| + C.$$

$$\frac{A}{t+4} + \frac{B}{t-1} = \frac{1}{(t+4)(t-1)}$$

$$\Rightarrow A(t-1) + B(t+4) = 1$$

$$\text{when } t=1: 5B=1 \Rightarrow B=1/5$$

$$\text{when } t=-4: -5A=1 \Rightarrow A=-1/5$$

24)

$$b) \int \frac{x^2}{(x-3)(x+2)^2} dx$$

$$= \int \left(\frac{9/25}{x-3} + \frac{16/25}{x+2} + \frac{-4/5}{(x+2)^2}\right) dx$$

$$= 9/25 \ln |x-3| + 16/25 \ln |x+2| + 4/5 \cdot (x+2)^{-1} + C.$$

$$\frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{x^2}{(x-3)(x+2)^2}$$

$$\Rightarrow A(x+2)^2 + B(x-3)(x+2) + C(x-3) = x^2$$

$$\text{when } x=3: 25A=9 \Rightarrow A=9/25$$

$$\text{when } x=-2: -5C=4 \Rightarrow C=-4/5$$

$$\text{when } x=0: 4A-6B-3C=0$$

$$\Rightarrow B = \frac{2}{5}A - \frac{1}{2}C = 16/25$$

28)

$$\int \frac{x^3}{(x+1)^3} dx$$

$$\int \left(1 - \frac{1}{x+1}\right)^3 dx$$

$$\int \left(1 - \frac{3}{x+1} + \frac{3}{(x+1)^2} - \frac{1}{(x+1)^3}\right) dx$$

$$x - 3 \ln|x+1| - 3(x+1)^{-1} + \frac{1}{2} \cdot (x+1)^{-2} + C$$

32)

$$\int \frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)} dx$$

$$\int \left(\frac{1}{x-1} + \frac{-1}{(x-1)^2} + \frac{-x+1}{x^2+1} \right) dx$$

~~$$\ln|x-1| + (x-1)^{-1} + \frac{1}{2} \ln|x^2+1| + \tan^{-1}(x) + C$$~~

$$\ln|x-1| + (x-1)^{-1} - \frac{1}{2} \ln|x^2+1| + \tan^{-1}(x) + C$$

42)

$$\int \frac{x^4 + 1}{x(x^2+1)^2} dx$$

$$= \int \frac{(x^2+1)^2 - 2x^2}{x(x^2+1)^2} dx$$

$$= \int \left(\frac{1}{x} - \frac{2x}{(x^2+1)^2} \right) dx$$

$$= \ln|x| + \frac{1}{x^2+1} + C$$

OR: use partial fractions with

$$\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} = \frac{x^3}{(x+1)^3}$$

$$\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1} = \frac{x^2-2x-1}{(x-1)^2(x^2+1)}$$

$$\Rightarrow A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2 = x^2-2x-1$$

when $x=1$: $2B = -2 \Rightarrow B = -1$

when $x=0$: $-A + B + D = -1 \Rightarrow -A + D = 0$

when $x=2$: $5A + 5B + 4C + 4D = -1$

when $x=-1$: $-4A + 2B - 4C + 4D = 2$

$$\Rightarrow A=1, C=-1, D=1$$