§7.2: 2, 10, 28, 34, 42, (52), 58

2. $\int \sin^6 x \cos^3 x \, dx = \int \sin^6 x \cos^2 x \cos x \, dx = \int \sin^6 x (1-\sin^2 x) \cos x \, dx$

(let $u = \sin x \rightarrow du = \cos x \, dx$)

$= \int u^6 (1-u^2) \, du = \int (u^6 - u^8) \, du = \frac{1}{7} u^7 - \frac{1}{9} u^9 + C$

$= \frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C$

10. $\int \sin^6 \pi x \, dx = \int (\sin^2 \pi x)^3 dx = \int \left[\frac{1}{2}(1 + \cos 2\pi x)\right]^3 dx$

$= \frac{1}{8} \int \left[1 - 3\cos 2\pi x + 3\cos^2 2\pi x - \cos^3 2\pi x\right] dx$

$= \frac{1}{8} \int \left[1 - 3\cos 2\pi x + \frac{3}{2}(1+\cos 4\pi x) - (1-\sin^2 2\pi x)\cos 2\pi x\right] dx$

$= \frac{1}{8} \int \left[\frac{5}{2} - 4\cos 2\pi x + \frac{3}{2}\cos 4\pi x + \sin^2 2\pi x \cos 2\pi x\right] dx$

$= \frac{1}{8}\left[\frac{5}{2} x - \frac{4}{2\pi} \sin 2\pi x + \frac{3}{8\pi} \sin 4\pi x + \frac{1}{3 \cdot 2\pi} \sin^3 2\pi x\right] + C$

$= \frac{5x}{16} - \frac{1}{4\pi} \sin 2\pi x + \frac{3}{64\pi} \sin 4\pi x + \frac{1}{48\pi} \sin^3 2\pi x + C$

28. $\int \tan^3 x \sec^3 x \, dx$ (let $u = \sec x \rightarrow du = \sec x \tan x \, dx$)

$= \int \sec^2 x \tan^2 x \sec x \tan x \, dx = \int u^2 (u^2 - 1) \, du = \int (u^4 - u^2) \, du$

$= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$

34. $\int \tan^2 x \sec x \, dx = \int (\sec^2 x - 1) \sec x \, dx = \int \sec^3 x \, dx - \int \sec x \, dx$

$= \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) - \ln|\sec x + \tan x| + C$ by example 8 etc.

$= \frac{1}{2}(\sec x \tan x - \ln|\sec x + \tan x|) + C$

42. $\int \sin(3x)\cos x\,dx = \int \frac{1}{2}[\sin(3x+x) + \sin(3x-x)]\,dx$

$= \frac{1}{2}\int (\sin(4x) + \sin(2x))\,dx$

$= -\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C$

52. $\int \sin x \cos x\,dx$

a. Substitution: Let $u = \cos x$. $du = -\sin x\,dx$

$\int \sin x \cos x\,dx = -\int u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}\cos^2 x + C_1$

b. Substitution: Let $u = \sin x$. $du = \cos x\,dx$

$\int \sin x \cos x\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C_2$

c. $\int \sin x \cos x\,dx = \int \frac{\sin 2x}{2}\,dx = -\frac{1}{4}\cos 2x + C_3$

d. Let $u = \sin x$ and $dv = \cos x\,dx$. $\rightarrow du = \cos x\,dx$ and $v = \sin x$

$\int \sin x \cos x\,dx = \sin^2 x - \int \sin x \cos x\,dx \rightarrow \int \sin x \cos x\,dx = \frac{1}{2}\sin^2 x + C_4$

These answers all differ by constants since

$\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

$\rightarrow \frac{-1}{4}\cos 2x = \frac{1}{2}\sin^2 x - \frac{1}{4} = -\frac{1}{2}\cos^2 x + \frac{1}{4}$

58. $\text{Vol} = \int_0^{\pi/4} \pi(\tan^2 x)^2\,dx = \pi\int_0^{\pi/4} \tan^2 x(\sec^2 x - 1)\,dx$

$= \pi\int_0^{\pi/4} \tan^2 x \sec^2 x\,dx - \pi\int_0^{\pi/4} \tan^2 x\,dx$ (Let $u = \tan x$ and $du = \sec^2 x\,dx$)

$= \pi\int_0^{\pi/4} u^2\,du - \pi\int_0^{\pi/4} (\sec^2 x - 1)\,dx$

$= \pi\left[\frac{u^3}{3}\right]_0^{\pi/4} - \pi\left[\tan x - x\right]_0^{\pi/4} = \pi\left[\frac{1}{3}\tan^3 x - \tan x + x\right]_0^{\pi/4}$

$= \pi\left[\frac{1}{3} - 1 + \frac{\pi}{4}\right] = \pi\left(\frac{\pi}{4} - \frac{2}{3}\right)$