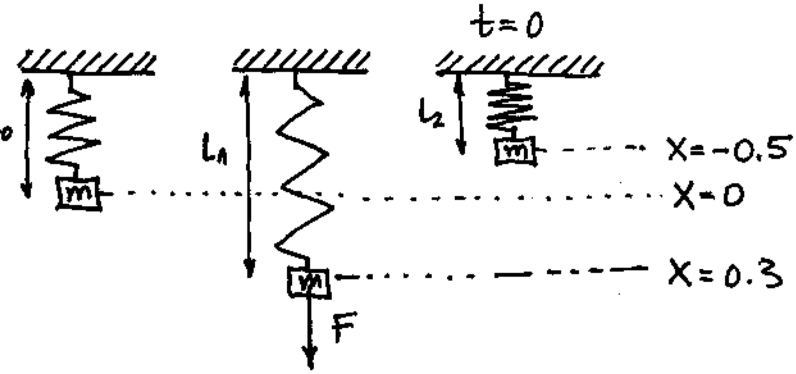
SECTION 17.3 # 2, 4a, 6, 16a,b,d,e

(2) $m = 4kg$
$l_0 = 1m$
$l_1 = 1.3\ m$
$F = 24.3\ N$
$l_2 = 0.8m$



By Hooke's law, $F = k \cdot \Delta l$ where k is the spring constant and Δl the change in length caused by force F.

$$k = \frac{F}{\Delta l} = \frac{F}{l_1 - l_0} = \frac{24.3N}{0.3m}$$

$$\underline{\underline{k = 81\ \frac{N}{m}}}$$

IF WE IGNORE ANY EXTERNAL RESISTING FORCES, THE MOTION OF THE MASS IS DESCRIBED BY

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$mx'' + kx = 0$$
$$4x'' + 81x = 0$$
$$4r^2 + 81 = 0 \quad \text{SO} \quad r = \pm \frac{9}{2} i$$

GENERAL SOLUTION: $x(t) = c_1 \cos\left(\frac{9}{2}t\right) + c_2 \sin\left(\frac{9}{2}t\right)$.
WE KNOW THAT AT TIME $t=0$ POSITION OF THE SPRING IS -0.5 m (SEE PICTURE). SO OUR FIRST INITIAL CONDITION IS $x(0) = -0.5$

$$x(0) = \underline{\underline{c_1 = -0.5}}$$

ALSO, SPRING IS RELEASED WITH ZERO VELOCITY, SO THE

SECOND INITIAL CONDITION IS $X'(0)=0$:

$$X'(t) = -\frac{9}{2}c_1 \sin\left(\frac{9}{2}t\right) + \frac{9}{2}c_2\cos\left(\frac{9}{2}t\right)$$

$$X'(0) = \frac{9}{2}C_2 = 0 \quad \text{SO} \quad \underline{\underline{C_2 = 0}}$$

~~PA~~ POSITION OF THE SPRING AT ANY TIME t IS THUS DESCRIBED BY

$$\boxed{X(t) = -0.5\cos\left(\frac{9}{2}t\right)}$$

(4) a) $m = 3\text{kg}$
$C = 30$
$K = 123$

IF WE TAKE THE FRICTIONAL FORCE INTO ACCOUNT, MOTION OF THE SPRING CAN BE DESCRIBED BY THE FOLLOWING EQUATION:

$$mX'' + CX + KX = 0$$

IN OUR CASE, $3X'' + 30X' + 123X = 0$

$$3r^2 + 30r + 123 = 0$$

$$r^2 + 10r + 41 = 0$$

$$r = \frac{-10 \pm \sqrt{100-164}}{2} = \frac{-10 \pm \sqrt{-64}}{2} = -5 \pm 4i$$

GENERAL SOLUTION: $X(t) = e^{-5t}(c_1\cos 4t + c_2 \sin 4t)$.

MOTION STARTS FROM EQUILIBRIUM POSITION, SO OUR FIRST INITIAL CONDITION IS

$X(0) = 0$

$$X(0) = \underline{\underline{C_1 = 0}}$$

INITIAL VELOCITY IS $2\,m/s$ SO THE SECOND INITIAL

CONDITION IS $x'(0)=2$

$$x'(t) = e^{-5t}(-4c_1\sin 4t + 4c_2\cos 4t) - 5e^{-5t}(c_1\cos 4t + c_2\sin 4t)$$

$$x'(0) = 4c_2 - 5c_1 = 2$$

$$4c_2 - 5\cdot 0 = 4c_2 = 2 \quad \text{SO} \quad \underline{\underline{c_2 = 1/2}}$$

THE MOTION OF THE SPRING IS DESCRIBED BY

$$\boxed{x(t) = \tfrac{1}{2}e^{-5t}\sin 4t}$$

(6) FOR CRITICAL DAMPING WE NEED

$$c^2 - 4mk = 0$$

$$c^2 = 4mk$$

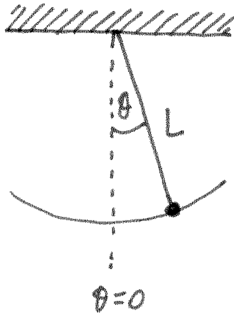
$$c = 2\sqrt{mk}$$

$$\boxed{c = 6\sqrt{41}}$$

FROM #4 $m=3$ AND $k=123$ SO $c = 2\sqrt{369} = 6\sqrt{41}$

$k = 3\cdot 41$

(16)



$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

FOR SMALL ANGLES θ $\sin\theta \approx \theta$, SO

$$\theta'' + \frac{g}{L}\theta = 0$$

a) $L = 1\,m$

$\theta(0) = 0.2$ rad

$\theta'(0) = 1$ rad/s

$L = 1\,m$ so $\theta'' + g\theta = 0$

$g \approx 9.81\ m/s^2$ so $\theta'' + 9.81\theta = 0$

$r^2 + 9.81 = 0 \qquad r = \pm i\sqrt{9.81}$

GENERAL SOLUTION: $\theta(t) = c_1 \cos(\sqrt{9.81}\,t) + c_2 \sin(\sqrt{9.81}\,t)$

INITIAL CONDITIONS:

I $\theta(0) = 0.2$

$$\theta(0) = \underline{\underline{c_1 = 0.2}}$$

II $\theta'(0) = 1$

$$\theta'(t) = -\sqrt{9.81}\, c_1 \sin(\sqrt{9.81}\,t) + \sqrt{9.81}\, c_2 \cos(\sqrt{9.81}\,t)$$

$$\theta'(0) = \sqrt{9.81}\, c_2 = 1$$

$$\underline{\underline{c_2 = 1/\sqrt{9.81}}}$$

$$\boxed{\theta(t) = 0.2 \cos(\sqrt{9.81}\,t) + \frac{1}{\sqrt{9.81}} \sin(\sqrt{9.81}\,t)}$$

b) TO FIND A MAXIMUM ANGLE, SET $\theta'(t) = 0$

$$-\sqrt{9.81}\, c_1 \sin(\sqrt{9.81}\,t) + \sqrt{9.81}\, c_2 \cos(\sqrt{9.81}\,t) = 0$$

$$\tan(\sqrt{9.81}\,t) = \frac{c_2}{c_1}$$

$$\sqrt{9.81}\,t = \arctan\frac{c_2}{c_1} + n\pi$$ where n is an integer.

TAKE ANY n, SAY $n = 0$. THEN

$$t = \frac{\arctan c_2/c_1}{\sqrt{9.81}}$$

$$\theta(t) = 0.2 \cos(\arctan c_2/c_1) + \frac{1}{\sqrt{9.81}} \sin(\arctan c_2/c_1)$$

$$\boxed{\theta(t) \approx 0.378 \text{ rad}}$$

(d) THE PENDULUM IS VERTICAL WHEN $\theta = 0$

$\theta(t) = 0$

$$0.2\cos(\sqrt{9.81}\,t) + \frac{1}{\sqrt{9.81}}\sin(\sqrt{9.81}\,t) = 0$$

$$\tan(\sqrt{9.81}\,t) = -0.2\cdot\sqrt{9.81}$$

$$\sqrt{9.81}\,t = \arctan(-0.2\sqrt{9.81}) + n\pi, \ n \text{ integer}$$

WHEN PENDULUM IS VERTICAL FOR THE FIRST TIME, $n = 1$

$$\text{so} \quad t = \frac{\arctan(-0.2\sqrt{9.81}) + \pi}{\sqrt{9.81}} \approx \boxed{0.824\,s}$$

(e) WHEN PENDULUM IS VERTICAL FIRST TIME, $t = 0.824s$.

$$\theta'(0.824) = -\sqrt{9.81}\cdot 0.2\sin(\sqrt{9.81}\cdot 0.824) + \cos(\sqrt{9.81}\cdot 0.824)$$
$$= -0.333 + (-0.847) \approx -1.1798 \text{ rad/s}$$

$$\boxed{\theta'(0.824) \approx -1.180 \text{ rad/s}}$$