SECTION 17.1 # (4), 8, 20, 30, 32

4. (OPTIONAL)

$$2y'' - y' - y = 0$$

THE CHARACTERISTIC EQUATION IS $2r^2 - r - 1 = 0$

$$r = \frac{1 \pm \sqrt{1+8}}{4} \quad \text{SO} \quad r_1 = 1 \quad \text{AND} \quad r_2 = -1/2$$

THE GENERAL SOLUTION IS THEN

$$\boxed{y_1 = c_1 e^{x} + c_2 e^{-1/2 x}}$$

8. $16y'' + 24y' + 9y = 0$

THE CHARACTERISTIC EQUATION IS $16r^2 + 24r + 9 = 0$

$$r = \frac{-24 \pm \sqrt{576-576}}{32} = -\frac{24}{32} = -\frac{3}{4}$$

THE GENERAL SOLUTION IS THEN

$$\boxed{y_1 = c_1 e^{-3/4 x} + c_2 x e^{-3/4 x}}$$

20. $y'' + 4y' + 6y = 0$
$y(0) = 2$
$y'(0) = 4$

THE CHARACTERISTIC EQUATION: $r^2 + 4r + 6 = 0$

$$r = \frac{-4 \pm \sqrt{16-24}}{2} = \frac{-4 \pm \sqrt{-8}}{2} = \frac{-4 \pm 2i\sqrt{2}}{2} = -2 \pm i\sqrt{2}$$

SO $\alpha = -2$ AND $\beta = \sqrt{2}$

THE GENERAL SOLUTION IS

$$y = e^{-2x}\left(c_1 \cos(\sqrt{2}x) + c_2 \sin(\sqrt{2}x)\right)$$

TO FIND PARTICULAR SOLUTION, WE SUBSTITUTE THE INITIAL CONDITIONS INTO THE GENERAL SOLUTION.

IST CONDITION: $y(0) = 2$

$$y = e^{0}\left(c_1 \cos 0 + c_2 \sin 0\right) = c_1 = 2 \qquad \text{SO } \underline{\underline{c_1 = 2}}$$

IIND CONDITION: $y'(0) = 4$

FIRST WE FIND y' FROM THE GENERAL SOLUTION:

$$y' = e^{-2x}\left[\sin(\sqrt{2}x)\cdot(-\sqrt{2}c_1 - 2c_2) + \cos(\sqrt{2}x)\cdot(\sqrt{2}c_2 - 2c_1)\right]$$

THEN WE SUBSTITUTE $y'(0) = 4$

$$y'(0) = \sqrt{2}c_2 - 2c_1 = 4$$

WE KNOW THAT $c_1 = 2$ SO

$$c_2 = \frac{4+2c_1}{\sqrt{2}} = \frac{4+4}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \underline{\underline{4\sqrt{2}}}$$

OUR SOLUTION BECOMES:

$$\boxed{y = e^{-2x}\left(2\cos(\sqrt{2}x) + 4\sqrt{2}\sin(\sqrt{2}x)\right)}$$

30. $y'' + 4y' + 3y = 0$
$y(1) = 0$
$y(3) = 2$

THE CHARACTERISTIC EQUATION IS $r^2 + 4r + 3 = 0$

$$r = \frac{-4 \pm \sqrt{16-12}}{2} \quad \text{SO} \quad r_1 = -3, \quad r_2 = -1$$

GENERAL SOLUTION: $y = c_1 e^{-3x} + c_2 e^{-x}$

WE USE THE BOUNDARY CONDITIONS TO FIND PARTICULAR SOLUTION:

1ST CONDITION

$y(1) = 0$

$$c_1 e^{-3} + c_2 e^{-1} = 0$$
$$e^{-1}(c_1 e^{-2} + c_2) = 0$$
$$e^{-1} \neq 0 \quad \text{SO} \quad c_1 e^{-2} + c_2 = 0$$
$$\underline{\underline{c_2 = -c_1 e^{-2}}}$$

IIND CONDITION

$y(3)=2$

$$c_1 e^{-9} + c_2 e^{-3} = 2$$

BUT WE KNOW $c_2 = -c_1 e^{-2}$

$$c_1 e^{-9} - c_1 e^{-5} = 2$$

$$c_1 (e^{-9} - e^{-5}) = 2$$

$$c_1 \left(\frac{1}{e^9} - \frac{1}{e^5} \right) = 2$$

$$c_1 \cdot \frac{1-e^4}{e^9} = 2$$

$$c_1 = \frac{2e^9}{1-e^4} \qquad c_2 = -\frac{2e^7}{1-e^4}$$

OUR SOLUTION IS THEN

$$\boxed{y = \frac{2e^9}{1-e^4} e^{-3x} - \frac{2e^7}{1-e^4} e^{-x}}$$

OR,

$$\boxed{y = \frac{1}{e^4-1} \left(-2e^{9-3x} + 2e^{7-x} \right)}$$

32. $y'' + 2y' + 5y = 0$

$y(0) = 1$

$y(\pi) = 2$

CHARACTERISTIC EQUATION: $r^2 + 2r + 5 = 0$

$$r = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

GENERAL SOLUTION:

$$y = e^{-x}(c_1 \cos 2x + c_2 \sin 2x)$$

WHEN WE SUBSTITUTE THE BOUNDARY CONDITIONS, WE GET:

I $y(0) = 1$

$$y(0) = e^0(c_1 \cos 0 + c_2 \sin 0) = \underline{\underline{c_1 = 1}}$$

II $y(\pi) = 2$

$$y(\pi) = e^{-\pi}(c_1 \cos 2\pi + c_2 \sin 2\pi) = c_1 e^{-\pi} = 2$$

$$\underline{\underline{c_1 = 2e^{\pi}}}$$

C_1 CANNOT BE BOTH 1 AND $2e^{\pi}$ SO WE CONCLUDE THAT SOLUTION TO THIS BOUNDARY PROBLEM DOES NOT EXIST.