11.9

4) $\dfrac{x}{1-x} = x + x^2 + x^3 + x^4 + \ldots$

Radius of Convergence $= 1$ Interval $= (-1, 1)$

12) $\dfrac{7x-1}{3x^2+2x-1} = \dfrac{1}{3x-1} + \dfrac{2}{x+1}$

$\dfrac{1}{3x-1} = -1 - 3x - 9x^2 - 27x^3 - \ldots$ $R = \frac{1}{3}$ $I = (-\frac{1}{3}, \frac{1}{3})$

$\dfrac{2}{x+1} = 2 - 2x + 2x^2 - 2x^3 + 2x^4 - \ldots$ $R = 1$ $I = (-1, 1)$

$\dfrac{7x-1}{3x^2+2x-1} = \displaystyle\sum_{n=0}^{\infty} \left[(-2)^n - 3^n\right] x^n$ Radius $= \frac{1}{3}$

Interval $= (-\frac{1}{3}, \frac{1}{3})$

14) $f(x) = x \ln(1+x)$

$= x^2 - \dfrac{x^3}{2} + \dfrac{x^4}{3} - \dfrac{x^5}{4} + \ldots = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+2}$

Radius $= 1$

26) $\displaystyle\int \frac{x}{1+x^5}\, dx = \int (x - x^6 + x^{11} - x^{16} + \ldots)\, dx$

$= \dfrac{x^2}{2} - \dfrac{x^7}{7} + \dfrac{x^{12}}{12} - \dfrac{x^{17}}{17} + \ldots = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{5n+2} x^{5n+2}$

30)

34) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!}$ $\quad f''(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-2}}{(2n-2)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n)!}$

$f(x) + f''(x) = 0$

39) $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$ Converges $-1<x<1$ by ratio test
$f(1) = \sum \frac{1}{n^2}$, $f(-1) = \sum \frac{(-1)^n}{n^2}$ both converge

Interval = $[-1, 1]$

$f'(x) = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n}$ Converges $-1<x<1$ by ratio test
$f'(1) = \sum_{n=0}^{\infty} \frac{1}{n+1}$ diverges

$f'(-1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ converges

Interval = $[-1, 1)$

$f''(x) = \sum_{n=1}^{\infty} \frac{n-1}{n} x^{n-2} = \sum_{n=2}^{\infty} \frac{n-1}{n} x^n$

Converges $-1<x<1$ by ratio test
$f''(1) = \sum \frac{n-1}{n}$ diverges (Divergence Test)

$f''(-1) = \sum \frac{n-1}{n}(-1)^n$ diverges (Divergence Test)

Interval = $(-1, 1)$