11.8

6) By the ratio test, the series converges when $\lim_{n\to\infty}\left|\frac{\frac{x^{n+1}}{(n+1)^2}}{\frac{x^n}{n^2}}\right| = \lim_{n\to\infty}|x|\cdot\frac{n^2}{(n+1)^2} = |x|$ <1. If $x=1$, the series is $\sum\frac{1}{n^2}$, which converges by the p-test.

If $x=-1$, the series converges absolutely.

So the radius of convergence is 1 and the interval of convergence is $[-1,1]$.

10) $\sum_{n=1}^{\infty}\frac{x^n}{n\cdot 3^n}$

By the ratio test, series converges when $\left|\frac{x}{3}\right|<1$, or when $-3<x<3$.

When $x=3$, series is $\sum\frac{1}{n}$, which diverges; when $x=-3$, series is $\sum\frac{(-1)^n}{n}$, which converges.

Radius $=3$; Interval $=[-3,3)$

(14) $\sum n^3(x-5)^n$

~~By the ratio test~~ $\cancel{\lim_{n\to\infty} n^3(x-5)^n}$

By the ratio test, series converges when $|x-5|<1$, or when $4<x<6$.

When $x=4$ or $x=6$, the terms of the series fail to converge to 0.

Radius $=1$; Interval $=(4,6)$.

18) $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

converges when $2|x+3| < 1$, or when $-7/2 < x < -5/2$

When $x = -7/2$, series is $\sum \frac{1}{\sqrt{n}}$, which diverges

When $x = -5/2$, series is $\sum \frac{(-1)^n}{\sqrt{n}}$, which converges

Interval $= (-7/2, -5/2]$ Radius $= 1/2$

24) $\sum_{n=1}^{\infty} \frac{nx^n}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$ converges for all x, by Ratio Test

(30) $\sum_{n=0} c_n x^n$ has radius of converge R, $4 \leq R < 6$
about 0

So it converges when $x = 1, -3$
diverges when $x = 8, -9$

(35) $f(x) = 1 + 2x + x^2 + 2x^3 + \ldots$

$= (1+2x)(1 + x^2 + x^4 + \ldots)$

$= \frac{1+2x}{1-x^2}$ Interval $= (-1, 1)$

36) $f(x) = \sum_{n=0}^{\infty} c_n x^n$ where $c_{n+4} = c_n$

$= (c_0 + c_1 x + c_2 x^2 + c_3 x^3)(1 + x^4 + x^8 + \ldots)$

$= \frac{c_0 + c_1 x + c_2 x^2 + c_3 x^3}{1 - x^4}$ Interval $= (-1, 1)$