11.7

6) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$

Since $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n < \sum_{n=1}^{\infty}\left(\frac{3n}{8n}\right)^n$, which converges,

the series converges. (Or use Root Test)

(12) $\displaystyle\sum_{n=1}^{\infty} \sin n$

Since $\displaystyle\lim_{n\to\infty} \sin n$ does not exist, series diverges

16) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$

Terms alternate, decrease, and tend to 0, so series converges.

However, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ diverges (use Limit Comparison Test with $\sum\frac{1}{\sqrt{n}}$)

so we only have conditional convergence.

(18) $\displaystyle\sum_{k=1}^{\infty} \frac{k+5}{5^k}$

Apply Ratio Test: $\displaystyle\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{n+6}{5^{n+1}}\cdot\frac{5^n}{n+5}\right| \to \frac{1}{5}$ as $n\to\infty$

So series converges

29) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2+4n}$

Since $\displaystyle\sum_{n=1}^{\infty} \frac{|\cos(n/2)|}{n^2+4n} < \sum_{n=1}^{\infty} \frac{1}{n^2}$; we have absolute convergence.

30) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

Since $\displaystyle\lim_{n\to\infty} \left(\frac{e^{1/n}}{n^2}\right)/\left(1/n^2\right) = \lim_{n\to\infty} e^{1/n} = 1$, we have

convergence by Limit Comparison Test (compared to $\sum \frac{1}{n^2}$)

34) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$

This series is convergent by the Root Test: $\displaystyle\lim_{n\to\infty} \sqrt[n]{\frac{(2n)^n}{n^{2n}}} = \frac{2n}{n^2} = \frac{2}{n} = 0.$

(36) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$

Since $(\ln n)(\ln(\ln n)) \geq 2\ln n$ for sufficently large values of n,

$$e^{(\ln n)(\ln(\ln n))} \geq e^{2\ln n} = n^2 \text{ for large } n$$

$$(\ln n)^{\ln n} \geq n^2$$

$$\frac{1}{(\ln n)^{\ln n}} \leq \frac{1}{n^2} \quad \text{for large } n$$

Since $\sum \frac{1}{n^2}$ converges, $\sum \frac{1}{(\ln n)^{\ln n}}$ converges as well.