§11.2: 12, 18, (21), (23), 26, (30), 44, 52 [redacted], (62), (63)

12. $1 - \frac{3}{2} + \frac{9}{4} - \frac{27}{8} + \dots$ geo series with $a=1$, $r = \frac{-3}{2}$

Since $|r| = 3/2 \geq 1$ series diverges.

18. $\sum_{n=1}^{\infty} \left(\frac{1}{e^2}\right)^n \Rightarrow a = \frac{1}{e^2} = |r| < 1$ so converges to:

$$\frac{\frac{1}{e^2}}{1 - \frac{1}{e^2}} = \frac{1}{e^2 - 1}$$

26. $s_n = \sum_{i=1}^{n} \frac{2}{i^2+4i+3} = \sum_{i=1}^{n} \left(\frac{1}{i+1} - \frac{1}{i+3}\right)$

$$= \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+2}\right) + \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$ telesc. series.

$$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3} = \lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}\right) = 5/6$$

44. $\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n}$ geo series with $r = \frac{x+3}{2}$

converges $\Leftrightarrow |r| < 1 \Leftrightarrow \left|\frac{x+3}{2}\right| < 1 \Leftrightarrow$ [redacted] $-5 < x < -1$

Then sum of series: $\frac{1}{1 - \frac{x+3}{2}} = \frac{2}{2-x+3} = -\frac{2}{x+1}$

52.

a. Initially, ball falls H, rebounds rH, falls rH, rebounds $r^2H, \ldots$

Total distance $= H + 2rH + 2r^2H + \ldots = H(1+2r+2r^2+\ldots)$
$= H\left(1+2r(1+r+r^2+\ldots)\right)$
$= H\left[1+2r\left(\frac{1}{1-r}\right)\right] = H\left(\frac{1+r}{1-r}\right)$ meters

b. from ex. 3 §2.1 know ball falls $\frac{1}{2}gt^2$ m/t sec

g = grav accel

So ball falls h meters in $t = \sqrt{\frac{2h}{g}}$ sec.

Total travel time $= \sqrt{\frac{2H}{g}} + 2\sqrt{\frac{2H}{g}r} + 2\sqrt{\frac{2H}{g}r^2} + \ldots$

$= \sqrt{\frac{2H}{g}}\left[1+2\sqrt{r}+2\sqrt{r^2}+2\sqrt{r^3}+\ldots\right]$

$= \sqrt{\frac{2H}{g}}\left[1+2\sqrt{r}\left(\frac{1}{1-\sqrt{r}}\right)\right] = \sqrt{\frac{2H}{g}}\left(\frac{1+r}{1-r}\right)$

c. ball hits ground with $v = -\sqrt{2hg}$
rebounds with $v = k\sqrt{2hg}$

ball takes time $k\sqrt{\frac{2h}{g}}$ to reach top of bounce
where height $= k^2h$

$\frac{d^2y}{dt^2} = -g \Rightarrow v = \frac{dy}{dt} = v_0 - gt \Rightarrow y = y_0 + v_0t - \frac{1}{2}gt^2$

# of descent	time of descent	speed before bounce	speed after bounce	time of ascent	peak height
1	$\sqrt{2H/g}$	$\sqrt{2Hg}$	$k\sqrt{2Hg}$	$k\sqrt{2H/g}$	k^2H
2	$\sqrt{2k^2H/g}$	$\sqrt{2k^2Hg}$	$k\sqrt{2k^2Hg}$	$k\sqrt{2k^2H/g}$	k^4H
3	$\sqrt{2k^4H/g}$	$\sqrt{2k^4Hg}$	$k\sqrt{2k^4Hg}$	$k\sqrt{2k^4H/g}$	k^6H
⋮					

total travel time in seconds:

$$\sqrt{\frac{2H}{g}} + k\sqrt{\frac{2H}{g}} + k\sqrt{\frac{2H}{g}} + k^2\sqrt{\frac{2H}{g}} + k^2\sqrt{\frac{2H}{g}} + \cdots$$

$$= \sqrt{\frac{2H}{g}}\left(1+2k+2k^2+\cdots\right) \quad = \sqrt{\frac{2H}{g}}\left[1+2k(1+k+k^2+\cdots)\right]$$

$$= \sqrt{\frac{2H}{g}}\left(\frac{1+k}{1-k}\right)$$

<u>Alt method</u>: use part b: at top of bounce height is $k^2h = rh$ so $\sqrt{r} = k$.

<u>Optional Problems</u>:

21. $\sum_{n=1}^{\infty} \frac{n}{n+5}$ diverges since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+5} = 1 \neq 0$

23. converges. $s_n = \sum_{i=1}^{n} \frac{1}{i(i+2)} = \sum_{i=1}^{n}\left(\frac{1/2}{i} - \frac{1/2}{i+2}\right) = \frac{1}{2}\sum_{i=1}^{n}\left(\frac{1}{i} - \frac{1}{i+2}\right)$

telescoping series: $\left(1-\frac{1}{3}\right) + \left(\frac{1}{2}-\frac{1}{4}\right) + \left(\frac{1}{3}-\frac{1}{5}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right)$

$$= 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}$$

so $\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \frac{1}{2}\lim_{n\to\infty}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right) = \frac{1}{2}\cdot\frac{3}{2}$

$$= 3/4$$

30. $\lim_{n\to\infty} \ln\left(\frac{n}{2n+5}\right) = \lim_{n\to\infty}\left(\frac{1}{2+\frac{5}{n}}\right) = \ln\frac{1}{2} \neq 0$

so <u>diverges</u>

62. No. Ex: $\sum a_n = \sum n$
$\sum b_n = \sum(-n)$ both diverge

yet $\sum(a_n+b_n) = \sum 0$ which converges to 0.

63. partial sums $\{s_n\}$ form increasing seq since
$s_n - s_{n-1} = a_n > 0 \quad \forall n$

$\{s_n\}$ bounded since $s_n \leq 1000 \quad \forall n$

So by Thm 11.1.10 $\{s_n\}$ converges
ie $\sum a_n$ is convergent