

$$3. \int \ln(x^2-1) dx$$

$$= \int \ln((x+1)(x-1)) dx$$

$$= \int \ln|x+1| dx + \int \ln|x-1| dx \quad \text{We then do 2 integrations by parts}$$

$$u = \ln(x+1) \quad v = x \\ dv = \frac{1}{x+1} dx \quad dv = dx$$

$$u = \ln(x-1) \quad v = x \\ dv = \frac{1}{x-1} dx \quad dv = dx$$

$$uv - \int v du + uv - \int v du$$

$$x \ln|x+1| - \int \frac{x dx}{x+1} + x \ln|x-1| - \int \frac{x dx}{x-1} \quad \text{We then do 2 u-substitutions}$$

$$u = x+1 \quad x = u-1 \\ dv = dx$$

$$u = x-1 \quad x = u+1 \\ dv = dx$$

$$= \dots - \int \frac{u-1}{u} du + \dots - \int \frac{u+1}{u} du$$

$$= -\int 1 - \frac{1}{u} du \quad -\int 1 + \frac{1}{u} du$$

$$= -u + \ln u \quad -u - \ln u \quad \text{We substitute the x's back in.} \\ -(x+1) + \ln|x+1| \quad -(x-1) - \ln|x-1|$$

$$= x \ln|x+1| + \ln|x+1| - (x+1) + x \ln|x-1| - \ln|x-1| - (x-1) + C$$

$$= \boxed{x \ln|x+1| + \ln|x+1| + x \ln|x-1| - \ln|x-1| - 2x + C}$$