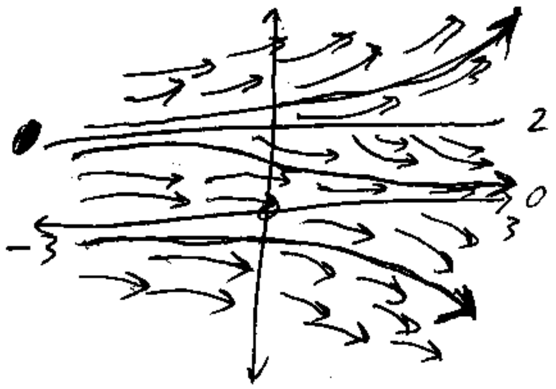
1.(i) The constant solutions to the equation $\frac{dx}{dt} = x^2(x-2)$ satisfy the equation $x^2(x-2)=0$ Thus, $x=0,2$ are the constant solutions.

(ii) $0<x<2$. The solution decreases in this region.

(iii) The direction field is as follows:



(iv) $\int dt = \int \frac{dx}{x^2(x-2)}$

$t = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} = \frac{1}{x^2(x-2)}$ Multiply by $x^2(x-2)$

$Ax(x-2)+B(x-2)+Cx^2=1$. Since this is true for all x, we substitute in $x=0,2$ for values of x.

For $x=0$, $-2B=1$

$B=-\frac{1}{2}$

$x=2$, $4C=1$

$C=\frac{1}{4}$. Use $x=1$ and these values of B and C.

$A(1)(1-2)+(-\frac{1}{2})(-1)+\frac{1}{4}(1)^2=1$

$-A=\frac{1}{4}$

$A=-\frac{1}{4}$ Thus

$\int \frac{dx}{x^2(x-2)} = \int(\frac{1}{4x} - \frac{1}{2x^2} + \frac{1}{4x-8})dx$

$= (\frac{1}{4}\int\frac{1}{x} - \frac{1}{2}\int\frac{1}{x^2} + \frac{1}{4}\int\frac{1}{x-2})dx$

$\boxed{t = -\frac{1}{4}\ln|x| + \frac{1}{2x} + \frac{1}{4}\ln|x-2| + C}$

(v) We substitute in the values $t=0$, $x=1$ as initial condition

$0 = -\frac{1}{4}\ln|1| + \frac{1}{2(1)} + \frac{1}{4}\ln|1-2| + C$

$= \frac{1}{2} + C$ $C = -\frac{1}{2}$

Thus, the equation is

$\boxed{t = -\frac{1}{4}\ln|x| + \frac{1}{2x} + \frac{1}{4}\ln|x-2| - \frac{1}{2}}$