SECTION 11.3 THE INTEGRAL TEST AND ESTIMATES OF SUMS

- The Integral Test - If f is positive, continuous, and decreasing on $[1,\infty)$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ is convergent if and only if $\int_1^{\infty} f(x)dx$ is convergent.
- The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p>1$ and divergent if $p\leq 1$.

- Remainder Estimate - If $\sum a_n$ converges by the Integral Test and $R_n = s - s_n$ then $\boxed{\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx.}$
- Since $s_n + R_n = s$, $\boxed{s_n + \int_{n+1}^{\infty} f(x)dx \leq s \leq s_n + \int_n^{\infty} f(x)dx}$

SECTION 11.4 THE COMPARISON TESTS

- The idea here is to compare a given series with a series with known convergence or divergence
- If $\sum a_n$ and $\sum b_n$ are series with positive terms,
  (i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.
  (ii) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.
- The Limit Comparison Test. If $\sum a_n$ and $\sum b_n$ are series with positive terms, then if $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where c is finite and $c>0$, then both series either converge or they both diverge.

SECTION 11.5 ALTERNATING SERIES

- These are useful for ~~terms~~ series whose terms are alternately positive and negative
- If $\sum_{n=1}^{\infty}(-1)^{n-1} b_n$ satisfies (i) $b_{n+1} \leq b_n$ for all n and (ii) $\lim_{n\to\infty} b_n = 0$, then the series is convergent. This is the Alternating Series Test.

- Alternating Series Estimation Theorem. If $s = \sum (-1)^{n-1} b_n$ is the sum of an alternating series that satisfies (i) $0 \leq b_{n+1} \leq b_n$ and (ii) $\lim_{n\to\infty} b_n = 0$, then $\boxed{|R_n| = |s - s_n| \leq b_{n+1}}$ The size of the error, in other words, is smaller than b_{n+1}, which is the absolute value of the first neglected term.