

## SECTION 9.7 PREDATOR-PREY SYSTEMS

- Without predators, the food supply would support the exponential growth of the prey. That is,  $\frac{dR}{dt} = kR$ , where  $k$  is a positive constant,  $R$  is the number of prey, and  $t$  is time.
  - Without prey, the predator population would decline at a rate proportional to itself. That is,  $\frac{dW}{dt} = -rW$ , where  $r$  is a positive constant,  $W$  is the number of predators, and  $t$  is time.
  - When both species interact, we assume that the principle cause of death for the prey is the predator, and birth and survival rates depend upon available food supply.
  - We assume the 2 species encounter each other at a rate proportional to both. Thus,  $\frac{dR}{dt} = kR - aRW$  and  $\frac{dW}{dt} = -rW + bRW$
- These are the predator-prey equations, or the Lotka-Volterra equations
- When the solutions of a system of differential equations are represented, the  $RW$ -plane is the phase plane, and the solution curves are phase trajectories.
  - When solving for  $\frac{dW}{dR}$  or  $\frac{dR}{dW}$ , use the Chain-rule to get rid of  $t$ .

## SECTION 11.1 INFINITE SEQUENCES AND SERIES

- A sequence can be thought of as a list of numbers in a definite order.
- The sequence  $\{a_1, a_2, a_3, \dots\}$  can be denoted by  $\{a_n\}$  or  $\{a_n\}_{n=1}^{\infty}$
- Sequences can be defined by giving a formula for the  $n^{\text{th}}$  term.
- The Fibonacci Sequence is defined recursively as having each term as the sum of the preceding 2 terms
- A sequence  $\{a_n\}$  has the limit  $L$ , where  $\lim_{n \rightarrow \infty} a_n = L$  or  $a_n \rightarrow L$  as  $n \rightarrow \infty$ .
- If for every  $\epsilon > 0$  there is a corresponding integer  $N$  such that  $|a_n - L| < \epsilon$  when  $n > N$ .
- If  $\lim_{n \rightarrow \infty} a_n$  exists, the sequence converges. Otherwise, it diverges.
- If  $\lim_{x \rightarrow \infty} f(x) = L$  and  $f(n) = a_n$  when  $n$  is an integer, then  $\lim_{n \rightarrow \infty} a_n = L$ .
- $\lim_{n \rightarrow \infty} a_n = \infty$  means for all positive numbers  $M$  there exists an integer  $N$  such that  $a_n > M$  when  $n > N$ .

### Limit Laws For Sequences

$$\lim_{n \rightarrow \infty} (a_n + b_n) = \lim_{n \rightarrow \infty} a_n + \lim_{n \rightarrow \infty} b_n$$

$$\lim_{n \rightarrow \infty} (a_n b_n) = \lim_{n \rightarrow \infty} a_n \cdot \lim_{n \rightarrow \infty} b_n$$

$$\lim_{n \rightarrow \infty} (a_n - b_n) = \lim_{n \rightarrow \infty} a_n - \lim_{n \rightarrow \infty} b_n$$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\lim_{n \rightarrow \infty} a_n}{\lim_{n \rightarrow \infty} b_n} \quad \text{if } \lim_{n \rightarrow \infty} b_n \neq 0$$

$$\lim_{n \rightarrow \infty} c a_n = c \lim_{n \rightarrow \infty} a_n$$

$$\lim_{n \rightarrow \infty} c = c$$