SECTION 9.5 THE LOGISTIC EQUATION

THE LOGISTIC DIFFERENTIAL EQUATION IS

$\boxed{\frac{dP}{dt} = kP\left(1-\frac{P}{K}\right)}$ where P = population, K = carrying capacity, t = time

- Thus, for small populations, the relative growth rate is almost constant. Also, if $P > K$, the relative growth rate $\frac{dP}{dt}$ is negative.
- See Page 616-617 for the derivation of the solution to the Logistic Equation
- The solution is $\boxed{P(t) = \frac{K}{1+Ae^{-kt}}}$, where $\boxed{A = \frac{K-P_0}{P_0}}$
- When comparing the Natural Growth and Logistic Models in modeling population data, for small values of t, both models are comparable in modeling the data. At a certain level of t, however, the exponential Natural Growth model becomes quite inaccurate.
- For species with a minimum population level m below which the species tends to become extinct, we use the modified version of the logistic equation:

$$\boxed{\frac{dP}{dt} = kP\left(1-\frac{P}{K}\right)\left(1-\frac{m}{P}\right)}$$

SECTION 9.6 LINEAR EQUATIONS

A FIRST-ORDER DIFFERENTIAL LINEAR EQUATION CAN BE PUT IN THE FORM

$\boxed{\frac{dy}{dx} + P(x)y = Q(x)}$, where P and Q are continuous functions on a given interval.

- See page 622-623 for the derivation of the solution to the linear differential equation. The solution is $\boxed{I(x) = e^{\int P(x)\,dx}}$, where $I(x)$ is the integrating factor.

- Thus, to solve the linear differential equation $y' + P(x)y = Q(x)$, multiply both sides by the integrating factor $I(x) = e^{\int P(x)dx}$ and integrate both sides