6) i) $F(x) = x \cdot \frac{1}{1-\frac{x}{3}} = x \cdot \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n = \boxed{\sum_{n=0}^{\infty} \frac{x^{n+1}}{3^n}}$

ii) Use Ratio Test.

$$L = \lim_{n\to\infty} \left|\frac{(n+2)\cdot x^{n+1}}{3^{n+1}}\right| \cdot \left|\frac{3^n}{(n+1)x^n}\right| = \lim_{n\to\infty} \left|\frac{n+2}{n+1}\right| \left|\frac{x}{3}\right| = \left|\frac{x}{3}\right|$$

$\left|\frac{x}{3}\right| < 1 \implies -1 < \frac{x}{3} < 1 \implies -3 < x < 3.$

So $\boxed{\text{Radius of Conv. is } 3.}$

iii) $F(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{3^n}$ by part i)

So $F'(x) = \sum_{n=0}^{\infty} \frac{(n+1)x^n}{3^n}$ as requested.

note: index of summation does not start at 1 because $F(x)$ has no constant term.

iv) $\sum_{n=0}^{\infty} \frac{n+1}{3^n} = F'(1)$

$F'(x) = \left[\frac{x}{1-\frac{x}{3}}\right]' = \frac{\left(1-\frac{x}{3}\right)\cdot 1 - x\left(-\frac{1}{3}\right)}{\left(1-\frac{x}{3}\right)^2} = \frac{1}{\left(1-\frac{x}{3}\right)^2}$ So $F'(1) = \frac{1}{\left(\frac{2}{3}\right)^2} = \boxed{\frac{9}{4}}$

7) $T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)\cdot(x-1)^n}{n!}$

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	$\arctan(x)$	$\pi/4$
1	$\frac{1}{1+x^2}$	$1/2$
2	$\frac{-2x}{(1+x^2)^2}$	$-1/2$
3	$\frac{(1+x^2)^2(-2)-(-2x)(2x)2(1+x^2)}{(1+x^2)^4}$	$\frac{2^2(-2)+2\cdot 2\cdot 2\cdot 2}{2^4} = \frac{8}{16} = \frac{1}{2}$

First four terms are $\frac{\pi}{4} + \frac{\frac{1}{2}(x-1)}{1!} \; \frac{-1(x-1)^2}{2\;\, 2!} + \frac{\frac{1}{2}(x-1)^3}{3!}$

$$\boxed{\frac{\pi}{4} + \frac{(x-1)}{2} - \frac{(x-1)^2}{4} + \frac{(x-1)^3}{12}}$$