SECTION 17.1 SECOND-ORDER LINEAR EQUATIONS

A second-order linear differential equation has the form

$$P(x)\frac{d^2y}{dx^2} + Q(x)\frac{dy}{dx} + R(x)y = G(x)$$

A homogeneous linear equation is one where $G(x)=0$.
A nonhomogeneous linear equation is one where $G(x)\neq 0$.

Theorem: If $y_1(x)$ and $y_2(x)$ are both solutions of the linear homogeneous equation (2) and c_1 and c_2 are any constants, then the function

$y(x) = c_1 y_1(x) + c_2 y_2(x)$ is also a solution of Equation 2.

Theorem: If y_1 and y_2 are linearly independent solutions of Equation 2, then the general solution is given by $y(x) = c_1 y_1(x) + c_2 y_2(x)$, where c_1 and c_2 are arbitrary constants

If the roots r_1 and r_2 of the auxiliary equation $ar^2+br+c=0$ are real and unequal, then the general solution of $ay''+by'+cy=0$ is

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

If the auxiliary equation $ar^2+br+c=0$ has only one real root r, then the general solution of $ay''+by'+cy=0$ is

$$y = c_1 e^{rx} + c_2 x e^{rx}$$

If the roots of the auxiliary equation $ar^2+br+c=0$ are the complex numbers $r_1 = \alpha + i\beta$, $r_2 = \alpha - i\beta$, then the general solution of $ay''+by'+cy=0$ is

$$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$$

SECTION 17.2 NONHOMOGENEOUS LINEAR EQUATIONS

THEOREM: THE General solution of the nonhomogeneous differential equation can be written as

$$y(x) = y_p(x) + y_c(x)$$

where y_p is a particular solution of Equation 1 and y_c is the general solution of the complementary Equation 2.

The Method of Undetermined Coefficients

For equation: $ay'' + by' + cy = G(x)$

where $G(x)$ is a polynomial. It is reasonable to guess that there is a particular solution y_p that is a polynomial of the same degree as G because if y is a polynomial then $ay'' + by' + cy$ is also a polynomial. We therefore substitute $y_p(x) =$ a polynomial (of the same degree as G) into the differential equation and determine the coefficients.

The Method of Variation Parameters

Suppose we have already solved the homogeneous equation $ay'' + by' + cy = 0$ and written the solution as

$$y(x) = c_1 y_1(x) + c_2 y_2(x)$$

where y_1 and y_2 are linearly independent solutions. Let's replace the constants c_1 and c_2 by arbitrary functions $v_1(x)$ and $v_2(x)$. We look for a particular solution of the nonhomogeneous equation $ay'' + by' + cy = G(x)$ of the form

$$y_p(x) = v_1(x) y_1(x) + v_2(x) y_2(x)$$

SECTION 17.3 APPLICATIONS OF SECOND-ORDER DIFFERENTIAL EQUATIONS

All are physics applications.

Vibrating Springs

$$x(t) = A\cos(\omega t + \delta)$$

$\omega = \sqrt{k/m}$ $A = \sqrt{c_1^2 + c_2^2}$ $\cos\delta = \frac{c_1}{A}$ $\sin\delta = -\frac{c_2}{A}$

Simple Harmonic Motion

Dampled Vibrations

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

c is the damping constant.

$c^2 - 4mk > 0$ Overdamping

Roots r_1 and r_2 are negative.

$c^2 - 4mk = 0$ Critical Damping

$r_1 = r_2 = -\frac{c}{2m}$

$c^2 - 4mk < 0$ Underdamping

$$\left.\begin{matrix} r_1 \\ r_2 \end{matrix}\right\} = \frac{c}{2m} \pm \omega i$$

$$\omega = \frac{\sqrt{4mk - c^2}}{2m}$$

Forced Vibrations

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

Resonance: Vibrations of Large amplitude.

Electric Circuits

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E(t)$$

SECTION 17.4 SERIES SOLUTIONS.

Use power series method, with a solution in the form

$$y = f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$$

Substitute this into the differential equation and determine the values of $c_0, c_1, c_2, \ldots$

Review the 2 Examples on p. 1146-1150

GOOD LUCK ON THE EXAM!