Name: _____ ID#: _____

Midterm II

Math S–1ab
Calculus I and II
Summer 2004

July 23, 2004

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

> *A student suspected of academic dishonesty in any form is subject to review and disciplinary action by the Summer School Administrative Board. Disciplinary action may include, but is not limited to, required withdrawal from the course and/or required withdrawal from the Summer School.* —*Handbook for Students*

Problem Number	Possible Points	Points Earned
1	10	
2	25	
3	15	
4	10	
5	15	
6	15	
7	10	
Total	100	

1 1

1. (10 Points) Gravel is being dumped from a conveyor belt at a rate of 30 cubic feet per minute and its coarseness is such that it forms a pile in the shape of a cone whose base is always twice its height. How fast is the height of the pile increasing when the pile is 10 feet high?

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2. (25 Points) Consider the function

$$f(x) = x^2 - 2x + \ln|x+2|.$$

(The page after the next is blank; a good place for scratch work!)

(i) What is the domain of f?

(ii) What asymptotes does the graph have, if any?

(iii) On what intervals is the function increasing? decreasing?

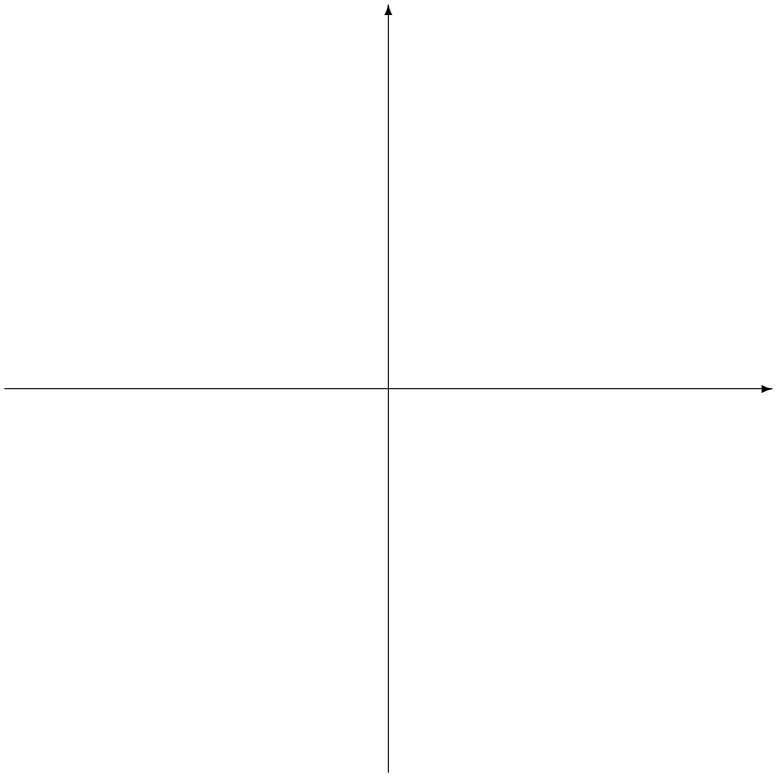
(iv) What is/are the critical point(s)? Give their x- and y-values. Label them as local or global maxima or minima.

(v) On what intervals is the function concave up? concave down?

(vi) Where are the inflection points, if any? Only give x-values since the y-values are quite complicated.

2 **2**

(vii) Sketch the graph of f, labeling the important points.



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2 2

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3 3

3. (15 Points) Find the following limits.

(i) $\lim_{x \to 0^+} \dfrac{\tan x}{1 - \cos x}$

(ii) $\lim_{x \to 0} \dfrac{x}{\cos x}$

3

3

(iii) $\lim_{x \to 0} \dfrac{\sqrt{1+x} - \frac{1}{2}x - 1}{x^2}$

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4 4

4. (10 Points) Find the slope of the line tangent to the curve

$$y^2 = x^3 + x$$

at the point $(1, \sqrt{2})$.

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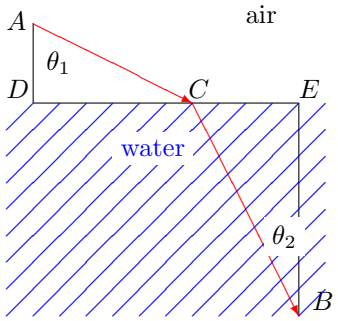
5 5

5. (15 Points) Let v_1 be the velocity of light in air and v_2 the velocity of light in water. According to Fermat's principle, a ray of light will travel from a point A to a point B in the water by a path ACB that minimizes the time taken. Show that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2},$$

where θ_1 (the angle of incidence) and θ_2 (the angle of refraction) are as shown. This equation is known as *Snell's Law*.



Hint: Let x be the distance DC, and w the distance DE. What x minimizes the time taken?

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6 6

6. (15 Points) Find the following indefinite integrals:

(i) $\int (x^2 + 3x + 5)\, dx$

(ii) $\int \frac{3}{\sqrt{1 - x^2}}\, dx$

(iii) $\int \frac{17}{x}\, \mathrm{dx}$

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7 7

7. (10 Points) Find $\int_0^1 4x(x^2+1)^9\,dx$.

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