

Name: _____ ID#: _____

Midterm I

Math S-1ab
Calculus I and II
Summer 2004

July 12, 2004

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

A student suspected of academic dishonesty in any form is subject to review and disciplinary action by the Summer School Administrative Board. Disciplinary action may include, but is not limited to, required withdrawal from the course and/or required withdrawal from the Summer School.

—Handbook for Students

Problem Number	Possible Points	Points Earned
1	20	
2	45	
3	10	
4	10	
5	15	
Total	100	

1**1**

1. (20 Points) Find the limits of the following functions, if they exist. If the limit does not exist, say so.

(i) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 4}{x - 4}$

(ii) $\lim_{x \rightarrow 0} \frac{2 - \cos x}{x^2 + 1}$

1

1

(iii) $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$

(iv) $\lim_{x \rightarrow \infty} \frac{1}{x} \cos x$

(v) $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + x}}{\sqrt{3x^2 - x}}$.

2**2**

2. (45 Points) Find the following derivatives:

(i) $\frac{d}{dx} (x^4 + 3x^3 + x + 2)$

(ii) $\frac{d}{d\theta} \frac{3}{1 + \cos \theta}$

2

2

(iii) $\frac{d}{dx} \sin\left(\frac{\pi}{5}\right)$

(iv) f' , where $f(t) = e^{2t} \sqrt{1+t^2}$

2

2

(v) g' , where $g(x) = \frac{\sin x}{x}$

(vi) h' , where $h(s) = \ln(\ln s)$

3

3

3. (10 Points) Find $\sqrt[3]{65}$ using a linear approximation.

4

4

4. (10 Points) Prove that right now there are two points on the equator on opposite sides of the world that have the exact temperature.

5. (15 Points) Define two new functions

$$\cosh x = \frac{e^x + e^{-x}}{2}; \quad \sinh x = \frac{e^x - e^{-x}}{2}.$$

(a) Prove that

$$\frac{d}{dx} \cosh x = \sinh x; \quad \frac{d}{dx} \sinh x = \cosh x.$$

(b) Let $\sinh^{-1} x$ be the inverse function to \sinh . Prove

$$\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}.$$

(This page intentionally left blank.)