Math 1A Fall 2001: Section 3.5 Solutions

2. Let $u = g(x) = 4 + 3x$ and $y = f(u) = \sqrt{u} = u^{1/2}$. Then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2}u^{-1/2}(3) = \frac{3}{2\sqrt{u}} = \frac{3}{2\sqrt{4+3x}}$.

6. Let $u = g(x) = e^x$ and $y = f(u) = \sin u$. Then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (\cos u)(e^x) = e^x \cos e^x$.

8. $F(x) = (x^2 - x + 1)^3 \Rightarrow F'(x) = 3(x^2 - x + 1)^2 (2x - 1)$

12. $y = a^3 + \cos^3 x \Rightarrow y' = 3(\cos x)^2(-\sin x)$ [a^3 is just a constant] $= -3\sin x \cos^2 x$

22. $s(t) = \sqrt[4]{\frac{t^3+1}{t^3-1}} = \left(\frac{t^3+1}{t^3-1}\right)^{1/4} \Rightarrow$

$$s'(t) = \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4} \frac{(t^3-1)(3t^2) - (t^3+1)(3t^2)}{(t^3-1)^2} = \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4} \frac{3t^2(t^3-1-t^3-1)}{(t^3-1)^2}$$

$$= \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4} \frac{3t^2(-2)}{(t^3-1)^2} = \frac{1}{2}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4} \frac{-3t^2}{(t^3-1)^2}$$

30. $y = \sqrt{x + \sqrt{x + \sqrt{x}}} \Rightarrow y' = \frac{1}{2}\left(x + \sqrt{x+\sqrt{x}}\right)^{-1/2}\left[1 + \frac{1}{2}(x+\sqrt{x})^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)\right]$

42. (a) $h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x))f'(x)$.
So $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$.

(b) $g(x) = f(x^2) \Rightarrow g'(x) = f'(x^2) \cdot \frac{d}{dx}(x^2) = f'(x^2)(2x)$.
So $g'(2) = f'(2^2)(2 \cdot 2) = 4f'(4) \approx 4(1.5) = 6$.

48. $f(x) = xg(x^2) \Rightarrow f'(x) = xg'(x^2)\,2x + g(x^2)\cdot 1 = 2x^2g'(x^2) + g(x^2) \Rightarrow$

$$f''(x) = 2x^2g''(x^2)\,2x + g'(x^2)\,4x + g'(x^2)\,2x = 4x^3g''(x^2) + 4xg'(x^2) + 2xg'(x^2)$$
$$= 6xg'(x^2) + 4x^3g''(x^2)$$

72. "The rate of change of y^5 with respect to x is eighty times the rate of change of y with respect to x" $\Leftrightarrow$ $\frac{d}{dx}y^5 = 80\frac{dy}{dx} \Leftrightarrow 5y^4\frac{dy}{dx} = 80\frac{dy}{dx} \Leftrightarrow 5y^4 = 80$ (Note that $dy/dx \neq 0$ since the curve never has a horizontal tangent) $\Leftrightarrow y^4 = 16 \Leftrightarrow y = 2$ (since $y > 0$ for all x)