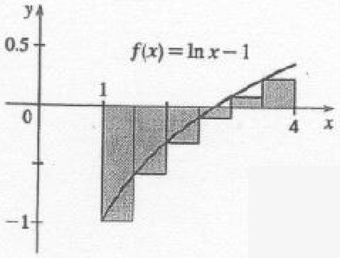
Math 1a Homework Solutions
Section 5.2

2. $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x \quad [x_i^* = x_{i-1}$ is a left endpoint and $\Delta x = 0.5]$

$$= 0.5\,[f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5)] \quad [f(x) = \ln x - 1]$$

$$\approx 0.5(-1 - 0.5945349 - 0.3068528 - 0.0837093 + 0.0986123 + 0.2527630)$$

$$= 0.5(-1.6337217) \approx -0.816861$$



The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the four rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.

8. (a) Using the right endpoints to approximate $\int_0^6 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(x_i)\,\Delta x = 2[f(2) + f(4) + f(6)] = 2(8.3 + 2.3 - 10.5) = 0.2$$

(b) Using the left endpoints to approximate $\int_0^6 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(x_{i-1})\,\Delta x = 2[f(0) + f(2) + f(4)] = 2(9.3 + 8.3 + 2.3) = 39.8$$

(c) Using the midpoint of each interval to approximate $\int_0^6 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(\overline{x}_i)\,\Delta x = 2[[f(1) + f(3) + f(5)]] = 2(9.0 + 6.5 - 7.6) = 15.8.$$

The estimate using the right endpoints must be less than $\int_0^6 f(x)\,dx$, since if we take x_i^* to be the right endpoint x_i of each interval, then $f(x_i) \le f(x)$ for all x on $[x_{i-1}, x_i]$, which implies that $f(x_i)\,\Delta x \le \int_{x_{i-1}}^{x_i} f(x)\,dx$, and so the sum $\sum_{i=1}^{3} [f(x_i)\,\Delta x] \le \sum_{i=1}^{3} \left[\int_{x_{i-1}}^{x_i} f(x)\,dx\right] = \int_0^6 f(x)\,dx$. Similarly, if we take x_i^* to be the left endpoint x_{i-1} of each interval, then $f(x_{i-1}) \ge f(x)$ for all x on $[x_{i-1}, x_i]$, and so $\sum_{i=1}^{3} [f(x_{i-1})\,\Delta x] \ge \int_0^6 f(x)\,dx$. We cannot say anything about the midpoint estimate.

12. $\Delta x = (4 - 2)/4 = 0.5$, so the endpoints are 2, 2.5, 3, 3.5, and 4, and the midpoints are 2.25, 2.75, 3.25, and 3.75. The Midpoint Rule gives

$$\int_2^4 x \ln x\,dx \approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x \quad [f(x) = x \ln x]$$

$$= 0.5[f(2.25) + f(2.75) + f(3.25) + f(3.75)] \approx 6.6969.$$

14. See the solution to Exercise 5.1.7 for a possible algorithm to calculate the sums. With $\Delta x = 0.01$ and subinterval endpoints $1, 1.01, 1.02, \ldots, 1.99, 2$, we calculate that the left Riemann sum is

$L_{100} = \sum_{i=1}^{100} \sqrt{1 + (x_{i-1})^2}\,\Delta x \approx 1.80598$, and the right Riemann sum is $R_{100} = \sum_{i=1}^{100} \sqrt{1 + (x_i)^2}\,\Delta x \approx 1.81420$.

Since $\sqrt{1 + x^2}$ is an increasing function, we must have $L_{100} \le \int_1^2 \sqrt{1 + x^2}\,dx \le R_{100}$, so

$1.805 < L_{100} \le \int_1^2 \sqrt{1 + x^2}\,dx \le R_{100} < 1.815$.

Therefore, the approximate value 1.8100 in Exercise 11 must be accurate to two decimal places.

36. $\int_0^3 |3x-5|\,dx$ can be interpreted as the area under the graph of the function $f(x) = |3x-5|$ between $x = 0$ and $x = 3$. This is equal to the sum of the areas of the two triangles, so $\int_0^3 |3x-5|\,dx = \frac{1}{2}\cdot\frac{5}{3}\cdot 5 + \frac{1}{2}\left(3-\frac{5}{3}\right)4 = \frac{41}{6}$.

42. $\int_0^1 f(t)\,dt + \int_1^3 f(t)\,dt + \int_3^4 f(t)\,dt = \int_0^4 f(t)\,dt \quad\Rightarrow\quad 2 + \int_1^3 f(t)\,dt + 1 = -6 \quad\Rightarrow$
$\int_1^3 f(t)\,dt = -6-2-1 = -9$

47. On $[1,3]$, $\ln 1 \le \ln x \le \ln 3 \quad\Rightarrow\quad 0(3-1) \le \int_1^3 \ln x\,dx \le (\ln 3)(3-1) \quad\Rightarrow\quad 0 \le \int_1^3 \ln x\,dx \le 2\ln 3$.