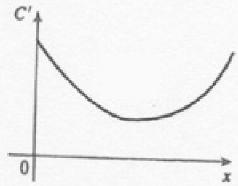
Math 1a Homework Solutions
Section 4.7

1. (a) $C(0)$ represents the fixed costs of production, such as rent, utilities, machinery etc., which are incurred even when nothing is produced.

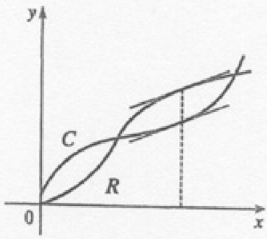
(b) The inflection point is the point at which $C''(x)$ changes from negative to positive; that is, the marginal cost $C'(x)$ changes from decreasing to increasing. Thus, the marginal cost is minimized at the inflection point.

(c) The marginal cost function is $C'(x)$. We graph it as in Example 1 in Section 2.8.

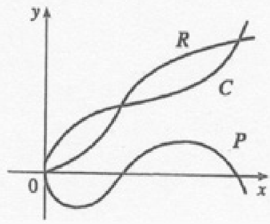


3. $c(x) = 21.4 - 0.002x$ and $c(x) = C(x)/x \;\Rightarrow\; C(x) = 21.4x - 0.002x^2$. $C'(x) = 21.4 - 0.004x$ and $C'(1000) = 17.4$. This means that the cost of producing the 1001st unit is about \$17.40.

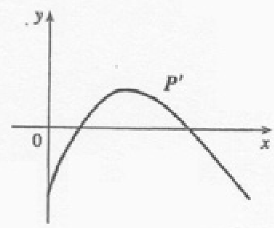
4. (a) Profit is maximized when the marginal revenue is equal to the marginal cost; that is, when R and C have equal slopes. See the box preceding Example 2.



(b) $P(x) = R(x) - C(x)$ is sketched.



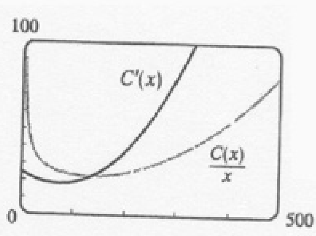
(c) The marginal profit function is defined as $P'(x)$.



8. (a) $C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3 \;\Rightarrow\; C'(x) = 25 - 0.18x + 0.0012x^2$ (marginal cost).

$c(x) = \dfrac{C(x)}{x} = \dfrac{339}{x} + 25 - 0.09x + 0.0004x^2$ (average cost).

(b)



The graphs intersect at $(135.56, 22.65)$, so the production level that minimizes average cost is about 136 units.

(c) $c'(x) = -\dfrac{339}{x^2} - 0.09 + 0.0008x = 0 \;\Rightarrow$

$x_1 \approx 135.56$. $c(x_1) \approx \$22.65/\text{unit}$.

(d) $C''(x) = -0.18 + 0.0024x = 0 \;\Rightarrow$

$x = \frac{1800}{24} = 75$. $C'(75) = \$18.25/\text{unit}$.

$C'''(x) = 0.0024 > 0$ for all x, so this is the minimum marginal cost.

14. (a) Cost = setup cost + manufacturing cost $\Rightarrow$ $C(x) = 500 + m(x) = 500 + 20x - 5x^{3/4} + 0.01x^2$. We can solve $x(p) = 320 - 7.7p$ for p in terms of x to find the demand (or price) function.

$x = 320 - 7.7p \Rightarrow 7.7p = 320 - x \Rightarrow p(x) = \dfrac{320 - x}{7.7}$. $R(x) = xp(x) = \dfrac{320x - x^2}{7.7}$.

(b) $C'(x) = R'(x) \Rightarrow 20 - \frac{15}{4}x^{-1/4} + 0.02x = \dfrac{320 - 2x}{7.7} \Rightarrow x \approx 81.53$ planes, and $p(x) = \$30.97$ million. The maximum profit associated with these values is about \$463.59 million.

16. (a) Let $p(x)$ be the demand function. Then $p(x)$ is linear and $y = p(x)$ passes through $(20, 10)$ and $(18, 11)$, so the slope is $-\frac{1}{2}$ and an equation of the line is $y - 10 = -\frac{1}{2}(x - 20) \Leftrightarrow y = -\frac{1}{2}x + 20$. Thus, the demand is $p(x) = -\frac{1}{2}x + 20$ and the revenue is $R(x) = xp(x) = -\frac{1}{2}x^2 + 20x$.

(b) The cost is $C(x) = 6x$, so the profit is $P(x) = R(x) - C(x) = -\frac{1}{2}x^2 + 14x$. Then $0 = P'(x) = -x + 14$ $\Rightarrow x = 14$. Since $P''(x) = -1 < 0$, the selling price for maximum profit is $p(14) = -\frac{1}{2}(14) + 20 = \$13$.

18. Let x denote the number of \$10 increases in rent. Then the price is $p(x) = 800 + 10x$, and the number of units occupied is $100 - x$. Now the revenue is

$$\begin{aligned} R(x) &= (\text{rental price per unit}) \times (\text{number of units rented}) \\ &= (800 + 10x)(100 - x) = -10x^2 + 200x + 80{,}000 \text{ for } 0 \le x \le 100 \quad \Rightarrow \end{aligned}$$

$R'(x) = -20x + 200 = 0 \Leftrightarrow x = 10$. This is a maximum since $R''(x) = -20 < 0$ for all x. Now we must check the value of $R(x) = (800 + 10x)(100 - x)$ at $x = 10$ and at the endpoints of the domain to see which value of x gives the maximum value of R. $R(0) = 80{,}000$, $R(10) = (900)(90) = 81{,}000$, and $R(100) = (1800)(0) = 0$. Thus, the maximum revenue of \$81,000/week occurs when 90 units are occupied at a rent of \$900/week.