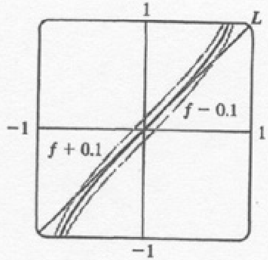
Math 1a Homework Solutions
Section 3.8

1. $f(x) = x^3 \Rightarrow f'(x) = 3x^2$, so $f(1) = 1$ and $f'(1) = 3$. With $a = 1$, $L(x) = f(a) + f'(a)(x - a)$ becomes $L(x) = f(1) + f'(1)(x - 1) = 1 + 3(x - 1) = 3x - 2$.

4. $f(x) = \sqrt[3]{x} = x^{1/3} \Rightarrow f'(x) = \frac{1}{3}x^{-2/3}$, so $f(-8) = -2$ and $f'(-8) = \frac{1}{12}$. Thus, $L(x) = f(-8) + f'(-8)(x + 8) = -2 + \frac{1}{12}(x + 8) = \frac{1}{12}x - \frac{4}{3}$.

8. $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$, so $f(0) = 0$ and $f'(0) = 1$.
Thus, $f(x) \approx f(0) + f'(0)(x - 0) = 0 + 1(x - 0) = x$.
We need $\tan x - 0.1 < x < \tan x + 0.1$, which is true when $-0.63 < x < 0.63$.



16. (a) The linear approximation of f at $a = 1$ is
$f(x) \approx f(1) + f'(1)(x - 1) = 2 + \sqrt{1^3 + 1}\,(x - 1) = 2 + \sqrt{2}\,(x - 1)$. So at $x = 1.1$,
$f(x) \approx 2 + 0.1\sqrt{2} \approx 2.1414$.

(b) The true value of $f(1.1)$ is greater than the linear estimate, since the derivative of the function is getting larger while the derivative of the approximation is constant.