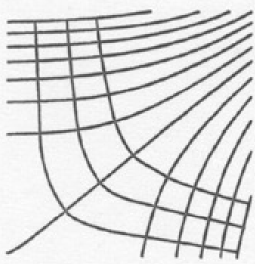
Math 1a Homework Solutions
Section 3.6 (II)

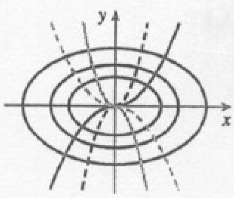
28. $y = (\sin^{-1} x)^2 \quad \Rightarrow \quad y' = 2(\sin^{-1} x)\dfrac{d}{dx}(\sin^{-1} x) \quad \Rightarrow \quad y' = \dfrac{2\sin^{-1} x}{\sqrt{1-x^2}}$

34. Let $y = \cos^{-1} x$. Then $\cos y = x$ and $0 \le y \le \pi \quad \Rightarrow \quad \dfrac{d}{dx}(\cos y) = \dfrac{d}{dx}(x) \quad \Rightarrow \quad -\sin y \dfrac{dy}{dx} = 1 \quad \Rightarrow$
$\dfrac{dy}{dx} = -\dfrac{1}{\sin y} = -\dfrac{1}{\sqrt{1-\cos^2 y}} = -\dfrac{1}{\sqrt{1-x^2}}$ (Note that $\sin y \ge 0$ for $0 \le y \le \pi$.)

40. The orthogonal family represents the direction of the wind.



44. $y = ax^3 \quad \Rightarrow \quad y' = 3ax^2$ and $x^2 + 3y^2 = b \quad \Rightarrow \quad 2x + 6yy' = 0 \quad \Rightarrow$
$3yy' = -x \quad \Rightarrow \quad y' = -\dfrac{x}{3(y)} = -\dfrac{x}{3(ax^3)} = -\dfrac{1}{3ax^2}$, so the curves are orthogonal.



50. $x^2 + 4y^2 = 36 \quad \Rightarrow \quad 2x + 8yy' = 0 \quad \Rightarrow \quad y' = -\dfrac{x}{4y}$. Let (a, b) be a point on $x^2 + 4y^2 = 36$ whose tangent line passes through $(12, 3)$. The tangent line is then $y - 3 = -\dfrac{a}{4b}(x - 12)$, so $b - 3 = -\dfrac{a}{4b}(a - 12)$. Multiplying both sides by $4b$ gives $4b^2 - 12b = -a^2 + 12a$, so $4b^2 + a^2 = 12(a + b)$. But $4b^2 + a^2 = 36$, so $36 = 12(a + b)$ $\Rightarrow \quad a + b = 3 \quad \Rightarrow \quad b = 3 - a$. Substituting $3 - a$ for b into $x^2 + 4y^2 = 36$ gives $a^2 + 4(3-a)^2 = 36 \quad \Leftrightarrow \quad a^2 + 36 - 24a + 4a^2 = 36 \quad \Leftrightarrow \quad 5a^2 - 24a = 0 \quad \Leftrightarrow \quad a(5a - 24) = 0$, so $a = 0$ or $a = \frac{24}{5}$. If $a = 0$, $b = 3 - 0 = 3$, and if $a = \frac{24}{5}$, $b = 3 - \frac{24}{5} = -\frac{9}{5}$. So the two points on the ellipse are $(0, 3)$ and $\left(\frac{24}{5}, -\frac{9}{5}\right)$. Using $y - 3 = -\frac{a}{4b}(x - 12)$ with $(a, b) = (0, 3)$ gives us the tangent line $y - 3 = 0$ or $y = 3$. With $(a, b) = \left(\frac{24}{5}, -\frac{9}{5}\right)$, we have $y - 3 = -\frac{24/5}{4(-9/5)}(x - 12) \quad \Leftrightarrow \quad y - 3 = \frac{2}{3}(x - 12) \quad \Leftrightarrow$ $y = \frac{2}{3}x - 5$. A graph of the ellipse and the tangent lines confirms our results.