Math 1a Homework Solutions
Section 3.6

1. (a) $\frac{d}{dx}\left(xy + 2x + 3x^2\right) = \frac{d}{dx}(4) \quad\Rightarrow\quad (x \cdot y' + y \cdot 1) + 2 + 6x = 0 \quad\Rightarrow\quad xy' = -y - 2 - 6x \quad\Rightarrow$
$y' = \dfrac{-y-2-6x}{x}$ or $y' = -6 - \dfrac{y+2}{x}$.

(b) $xy + 2x + 3x^2 = 4 \quad\Rightarrow\quad xy = 4 - 2x - 3x^2 \quad\Rightarrow\quad y = \dfrac{4 - 2x - 3x^2}{x} = \dfrac{4}{x} - 2 - 3x$, so $y' = -\dfrac{4}{x^2} - 3$.

(c) From part (a), $y' = \dfrac{-y-2-6x}{x} = \dfrac{-(4/x - 2 - 3x) - 2 - 6x}{x} = \dfrac{-4/x - 3x}{x} = -\dfrac{4}{x^2} - 3$.

8. $\sqrt{1 + x^2y^2} = 2xy \quad\Rightarrow\quad \frac{1}{2}\left(1 + x^2y^2\right)^{-1/2}\left(x^2 \cdot 2yy' + y^2 \cdot 2x\right) = 2(xy' + y \cdot 1) \quad\Rightarrow$

$$\frac{2x^2y}{2\sqrt{1+x^2y^2}}\,y' + \frac{2xy^2}{2\sqrt{1+x^2y^2}} = 2xy' + 2y \quad\Rightarrow\quad y'\left(\frac{x^2y}{\sqrt{1+x^2y^2}} - 2x\right) = 2y - \frac{xy^2}{\sqrt{1+x^2y^2}} \quad\Rightarrow$$

$$y'\left(\frac{x^2y - 2x\sqrt{1+x^2y^2}}{\sqrt{1+x^2y^2}}\right) = \frac{2y\sqrt{1+x^2y^2} - xy^2}{\sqrt{1+x^2y^2}} \quad\Rightarrow$$

$$y' = \frac{2y\sqrt{1+x^2y^2} - xy^2}{x^2y - 2x\sqrt{1+x^2y^2}} = \frac{y\left(2\sqrt{1+x^2y^2} - xy\right)}{x\left(xy - 2\sqrt{1+x^2y^2}\right)} = -\frac{y}{x}$$

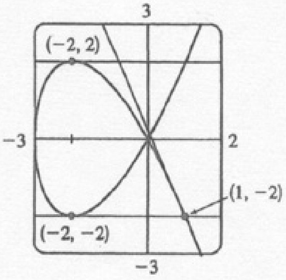
Another method: Since $1 + x^2y^2$ is positive, we can square both sides first and then differentiate implicitly.

16. $x^{2/3} + y^{2/3} = 4 \quad\Rightarrow\quad \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0 \quad\Rightarrow\quad \dfrac{1}{\sqrt[3]{x}} + \dfrac{y'}{\sqrt[3]{y}} = 0 \quad\Rightarrow\quad y' = -\dfrac{\sqrt[3]{y}}{\sqrt[3]{x}}$. When $x = -3\sqrt{3}$ and $y = 1$, we have $y' = -\dfrac{1}{\left(-3\sqrt{3}\right)^{1/3}} = -\dfrac{\left(-3\sqrt{3}\right)^{2/3}}{-3\sqrt{3}} = \dfrac{3}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}}$, so an equation of the tangent line is $y - 1 = \frac{1}{\sqrt{3}}\left(x + 3\sqrt{3}\right)$ or $y = \frac{1}{\sqrt{3}}x + 4$.

20. (a) $y^2 = x^3 + 3x^2 \quad\Rightarrow\quad 2yy' = 3x^2 + 3(2x) \quad\Rightarrow\quad y' = \dfrac{3x^2 + 6x}{2y}$. So at the point $(1, -2)$ we have
$y' = \dfrac{3(1)^2 + 6(1)}{2(-2)} = -\dfrac{9}{4}$, and an equation of the tangent line is $y + 2 = -\frac{9}{4}(x - 1)$ or $y = -\frac{9}{4}x + \frac{1}{4}$.

(b) The curve has a horizontal tangent where $y' = 0 \quad\Leftrightarrow$
$3x^2 + 6x = 0 \quad\Leftrightarrow\quad 3x(x+2) = 0 \quad\Leftrightarrow\quad x = 0$ or $x = -2$. But note that at $x = 0$, $y = 0$ also, so the derivative does not exist. At $x = -2$, $y^2 = (-2)^3 + 3(-2)^2 = -8 + 12 = 4$, so $y = \pm 2$. So the two points at which the curve has a horizontal tangent are $(-2, -2)$ and $(-2, 2)$.

(c)



24. $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1 \quad\Rightarrow\quad \dfrac{2x}{a^2}+\dfrac{2yy'}{b^2}=0 \quad\Rightarrow\quad y'=-\dfrac{b^2x}{a^2y}$ $\Rightarrow$ the equation of the tangent at (x_0, y_0) is $y-y_0=\dfrac{-b^2x_0}{a^2y_0}(x-x_0)$. Multiplying both sides by $\dfrac{y_0}{b^2}$ gives $\dfrac{y_0y}{b^2}-\dfrac{y_0^2}{b^2}=-\dfrac{x_0x}{a^2}+\dfrac{x_0^2}{a^2}$. Since (x_0, y_0) lies on the ellipse, we have $\dfrac{x_0x}{a^2}+\dfrac{y_0y}{b^2}=\dfrac{x_0^2}{a^2}+\dfrac{y_0^2}{b^2}=1$.

26. $x^2+6xy+y^2=8 \quad\Rightarrow\quad 2x+6xy'+6y+2yy'=0 \quad\Rightarrow\quad (3x+y)y'=-x-3y \quad\Rightarrow$

$$y'=-\frac{x+3y}{3x+y} \quad\Rightarrow$$

$$y''=-\frac{(3x+y)(1+3y')-(x+3y)(3+y')}{(3x+y)^2}=-\frac{-8y+8xy'}{(3x+y)^2}=\frac{8\,(y-xy')}{(3x+y)^2}$$

$$=\frac{8[y-x(-x-3y)/(3x+y)]}{(3x+y)^2}\cdot\frac{3x+y}{3x+y}=\frac{8[y(3x+y)+x(x+3y)]}{(3x+y)^3}$$

$$=\frac{8(x^2+6xy+y^2)}{(3x+y)^3}=\frac{64}{(3x+y)^3}$$

At the last step, we used the fact that x and y must satisfy the original equation, $x^2+6xy+y^2=8$.