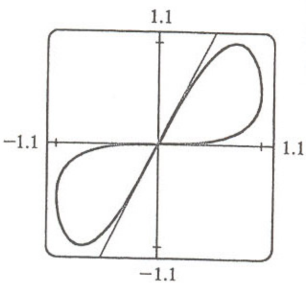
Section 3.5 (continued)

66. $x = \sin t$, $y = \sin(t + \sin t)$; $(0, 0)$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos(t+\sin t)(1+\cos t)}{\cos t} = \cos(t+\sin t)\,\frac{1+\cos t}{\cos t} = \cos(t+\sin t)\left(\frac{1}{\cos t}+1\right)$$
$$= (\sec t + 1)\cos(t + \sin t).$$



Now $x = \sin t$ is 0 when $t = 0$ and $t = \pi$, so there are two tangents at the point $(0, 0)$ since both $t = 0$ and $t = \pi$ correspond to the origin. The tangent corresponding to $t = 0$ has slope $(\sec 0 + 1)\cos(0 + \sin 0) = 2\cos 0 = 2$, and its equation is $y = 2x$. The tangent corresponding to $t = \pi$ has slope $(\sec\pi + 1)\cos(\pi + \sin\pi) = 0$, so it is the x-axis; that is, $y = 0$.

68. (a) $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$ $\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{r(\sin\theta)}{r(1-\cos\theta)} = \dfrac{\sin\theta}{1-\cos\theta}$. When $\theta = \frac{\pi}{3}$,

$\dfrac{dy}{dx} = \dfrac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = \sqrt{3}$, $(x, y) = \left(r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right), \frac{1}{2}r\right)$, and the tangent is $y - \frac{1}{2}r = \sqrt{3}\left[x - r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right]$.

(b) Horizontal tangent: $dy/dx = 0$ $\Leftrightarrow$ $\sin\theta = 0$ (and $\cos\theta \neq 1$) $\Leftrightarrow$ $\theta = (2n+1)\pi$. The corresponding points are $((2n+1)\pi r, 2r)$.

Vertical tangent: dy/dx is undefined $\Leftrightarrow$ $1 - \cos\theta = 0$ $\Leftrightarrow$ $\cos\theta = 1$ $\Leftrightarrow$ $\theta = 2n\pi$. The corresponding points are $(2n\pi r, 0)$.

(c)