

## Section 3.5

2. Let  $u = g(x) = 4 + 3x$  and  $y = f(u) = \sqrt{u} = u^{1/2}$ . Then  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{1}{2}u^{-1/2}(3) = \frac{3}{2\sqrt{u}} = \frac{3}{2\sqrt{4+3x}}$ .

6. Let  $u = g(x) = e^x$  and  $y = f(u) = \sin u$ . Then  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\cos u)(e^x) = e^x \cos e^x$ .

16.  $g(x) = e^{-5x} \cos 3x \Rightarrow g'(x) = e^{-5x}(-3 \sin 3x) + (\cos 3x)(-5e^{-5x}) = -e^{-5x}(3 \sin 3x + 5 \cos 3x)$

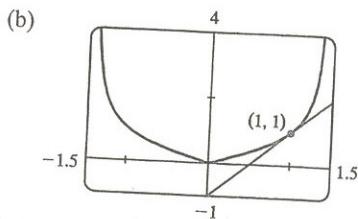
28.  $y = \sin(\sin(\sin x)) \Rightarrow y' = \cos(\sin(\sin x)) \frac{d}{dx}(\sin(\sin x)) = \cos(\sin(\sin x)) \cos(\sin x) \cos x$

34. (a) For  $x > 0$ ,  $|x| = x$ , and  $y = f(x) = \frac{x}{\sqrt{2-x^2}} \Rightarrow$

$$f'(x) = \frac{\sqrt{2-x^2}(1) - x(\frac{1}{2})(2-x^2)^{-1/2}(-2x)}{(\sqrt{2-x^2})^2} \cdot \frac{(2-x^2)^{1/2}}{(2-x^2)^{1/2}}$$

$$= \frac{(2-x^2) + x^2}{(2-x^2)^{3/2}} = \frac{2}{(2-x^2)^{3/2}}$$

So at  $(1, 1)$ , the slope of the tangent line is  $f'(1) = 2$  and its equation is  $y - 1 = 2(x - 1)$  or  $y = 2x - 1$ .



42. (a)  $h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x))f'(x)$ .

So  $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$ .

(b)  $g(x) = f(x^2) \Rightarrow g'(x) = f'(x^2) \cdot \frac{d}{dx}(x^2) = f'(x^2)(2x)$ .

So  $g'(2) = f'(2^2)(2 \cdot 2) = 4f'(4) \approx 4(1.5) = 6$ .

54.  $f(x) = xe^{-x}$ ,  $f'(x) = e^{-x} - xe^{-x} = (1-x)e^{-x}$ ,  $f''(x) = -e^{-x} + (1-x)(-e^{-x}) = (x-2)e^{-x}$ . Similarly,  $f'''(x) = (3-x)e^{-x}$ ,  $f^{(4)}(x) = (x-4)e^{-x}$ , ...,  $f^{(1000)}(x) = (x-1000)e^{-x}$ .

56. (a)  $s = A \cos(\omega t + \delta) \Rightarrow \text{velocity} = s' = -\omega A \sin(\omega t + \delta)$ .

(b) If  $A \neq 0$  and  $\omega \neq 0$ , then  $s' = 0 \Leftrightarrow \sin(\omega t + \delta) = 0 \Leftrightarrow \omega t + \delta = n\pi \Leftrightarrow t = \frac{n\pi - \delta}{\omega}$ ,  
 $n$  an integer.

58.  $L(t) = 12 + 2.8 \sin\left(\frac{2\pi}{365}(t-80)\right) \Rightarrow L'(t) = 2.8 \cos\left(\frac{2\pi}{365}(t-80)\right)\left(\frac{2\pi}{365}\right)$ .

On March 21,  $t = 80$ , and  $L'(80) \approx 0.0482$  hours per day. On May 21,  $t = 141$ , and  $L'(141) \approx 0.02398$ , which is approximately one-half of  $L'(80)$ .