Section 3.4

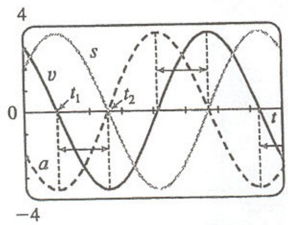
8. $y = \dfrac{\sin x}{1+\cos x} \quad \Rightarrow$

$$\frac{dy}{dx} = \frac{(1+\cos x)\cos x - \sin x\,(-\sin x)}{(1+\cos x)^2} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{\cos x + 1}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

20. (a) $y = \sec x - 2\cos x \quad \Rightarrow \quad y' = \sec x \tan x + 2\sin x \quad \Rightarrow$
the slope of the tangent line at $\left(\frac{\pi}{3}, 1\right)$ is
$\sec\frac{\pi}{3}\tan\frac{\pi}{3} + 2\sin\frac{\pi}{3} = 2\cdot\sqrt{3} + 2\cdot\frac{\sqrt{3}}{2} = 3\sqrt{3}$ and an equation is
$y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{3}\right)$ or $y = 3\sqrt{3}\,x + 1 - \pi\sqrt{3}$.

30. (a) $s(t) = 2\cos t + 3\sin t \quad \Rightarrow \quad v(t) = -2\sin t + 3\cos t \quad \Rightarrow \quad a(t) = -2\cos t - 3\sin t$

(b)

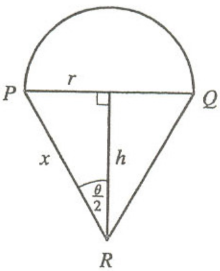


(c) $s = 0 \quad \Rightarrow \quad t_2 \approx 2.55$. So the mass passes through the equilibrium position for the first time when $t \approx 2.55$ s.

(d) $v = 0 \quad \Rightarrow \quad t_1 \approx 0.98$, $s(t_1) \approx 3.61$ cm. So the mass travels a maximum of about 3.6 cm (upward and downward) from its equilibrium position.

(e) The speed $|v|$ is greatest when $s = 0$; that is, when $t = t_2 + n\pi$, n a positive integer. The mass is speeding up when v and a have the same sign. From the figure, we see that this is the case on the intervals $(t_1 + n\pi, t_2 + n\pi)$ where n is a whole number.

42. Let $|PR| = x$. Then we get the following formulas for r and h in terms of θ and x:
$\sin\dfrac{\theta}{2} = \dfrac{r}{x} \quad \Rightarrow \quad r = x\sin\dfrac{\theta}{2}$ and $\cos\dfrac{\theta}{2} = \dfrac{h}{x} \quad \Rightarrow \quad h = x\cos\dfrac{\theta}{2}$. Now
$A(\theta) = \frac{1}{2}\pi r^2$ and $B(\theta) = \frac{1}{2}(2r)h = rh$. So



$$\lim_{\theta\to 0^+}\frac{A(\theta)}{B(\theta)} = \lim_{\theta\to 0^+}\frac{\frac{1}{2}\pi r^2}{rh} = \tfrac{1}{2}\pi\lim_{\theta\to 0^+}\frac{r}{h} = \tfrac{1}{2}\pi\lim_{\theta\to 0^+}\frac{x\sin(\theta/2)}{x\cos(\theta/2)}$$
$$= \tfrac{1}{2}\pi\lim_{\theta\to 0^+}\tan(\theta/2) = 0.$$