Section 3.3

1. (a) $s = f(t) = t^3 - 12t^2 + 36t \quad \Rightarrow \quad v(t) = f'(t) = 3t^2 - 24t + 36$

   (b) $v(3) = 27 - 72 + 36 = -9$ m/s

   (c) The particle is at rest when $v(t) = 0$. $3t^2 - 24t + 36 = 0 \quad \Rightarrow \quad 3(t-2)(t-6) = 0 \quad \Rightarrow \quad t = 2, 6$.

   (d) The particle is moving in the positive direction when $v(t) > 0$. $3(t-2)(t-6) > 0 \quad \Leftrightarrow \quad 0 \le t < 2$ or $t > 6$.

   (e) Since the particle is moving forward and backward, we need to calculate the distance traveled in the intervals $[0, 2]$, $[2, 6]$, and $[6, 8]$ separately.

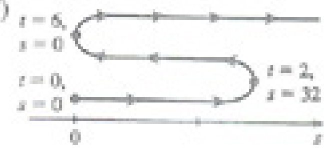
   $|f(2) - f(0)| = |32 - 0| = 32$.

   $|f(6) - f(2)| = |0 - 32| = 32$.

   $|f(8) - f(6)| = |32 - 0| = 32$.

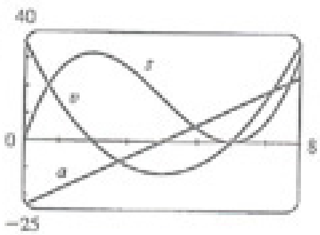
   The total distance is $32 + 32 + 32 = 96$ m.

   (f)



   (g) $a(t) = v'(t) = 6t - 24$. $a(3) = 6(3) - 24 = -6$ (m/s)/s or m/s^2.

   (h)



   (i) The particle is speeding up when v and a have the same sign. This occurs when $2 < t < 4$ and when $t > 6$. It is slowing down when v and a have opposite signs; that is, when $0 \le t < 2$ and when $4 < t < 6$.

12. $V(t) = 5000\left(1 - \frac{1}{40}t\right)^2 = 5000\left(1 - \frac{1}{20}t + \frac{1}{1600}t^2\right) \quad \Rightarrow \quad V'(t) = 5000\left(-\frac{1}{20} + \frac{1}{800}t\right) = -250\left(1 - \frac{1}{40}t\right)$

    (a) $V'(5) = -250\left(1 - \frac{5}{40}\right) = -218.75$ gal/min

    (b) $V'(10) = -250\left(1 - \frac{10}{40}\right) = -187.5$ gal/min

    (c) $V'(20) = -250\left(1 - \frac{20}{40}\right) = -125$ gal/min

    (d) $V'(40) = -250\left(1 - \frac{40}{40}\right) = 0$ gal/min

    The water is flowing out the fastest at the beginning — when $t = 0$, $V'(t) = -250$ gal/min. The water is flowing out the slowest at the end — when $t = 40$, $V'(t) = 0$. As the tank empties, the water flows out more slowly.

22. (a) (i) $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}} = \left(\frac{1}{2}\sqrt{\frac{T}{\rho}}\right)L^{-1} \quad \Rightarrow \quad \frac{df}{dL} = -\left(\frac{1}{2}\sqrt{\frac{T}{\rho}}\right)L^{-2} = -\frac{1}{2L^2}\sqrt{\frac{T}{\rho}}$

    (ii) $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}} = \left(\frac{1}{2L\sqrt{\rho}}\right)T^{1/2} \quad \Rightarrow \quad \frac{df}{dT} = \frac{1}{2}\left(\frac{1}{2L\sqrt{\rho}}\right)T^{-1/2} = \frac{1}{4L\sqrt{T\rho}}$

    (iii) $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}} = \left(\frac{\sqrt{T}}{2L}\right)\rho^{-1/2} \quad \Rightarrow \quad \frac{df}{d\rho} = -\frac{1}{2}\left(\frac{\sqrt{T}}{2L}\right)\rho^{-3/2} = -\frac{\sqrt{T}}{4L\rho^{3/2}}$

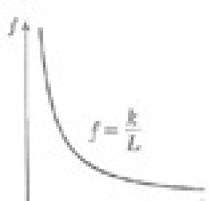
    (b) *Note:* Illustrating tangent lines on the generic figures may help to explain the results.

    (i) $\frac{df}{dL} < 0$ and L is decreasing $\Rightarrow$ f is increasing $\Rightarrow$ higher note

    (ii) $\frac{df}{dT} > 0$ and T is increasing $\Rightarrow$ f is increasing $\Rightarrow$ higher note

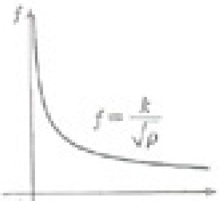
    (iii) $\frac{df}{d\rho} < 0$ and ρ is increasing $\Rightarrow$ f is decreasing $\Rightarrow$ lower note

(i)



(ii)

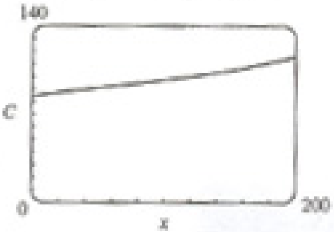
(iii)



24. (a) $C(x) = 84 + 0.16x - 0.0006x^2 + 0.000003x^3 \Rightarrow C'(x) = 0.16 - 0.0012x + 0.000009x^2 \Rightarrow$ $C'(100) = 0.13$. This is the rate at which the cost is increasing as the 100th item is produced.

(b) $C(101) - C(100) = 97.13030299 - 97 \approx \$0.13$.

(c)



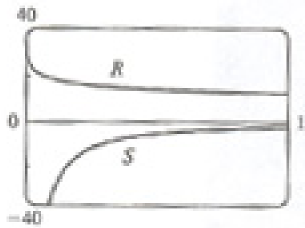
From the graph, we can estimate the x-coordinate of the point of inflection to be between 60 and 80.

(d) $C''(x) = -0.0012 + 0.000018x = 0 \Rightarrow$ $x = 66\frac{2}{3}$ and $C''(x)$ changes from negative to positive at this value of x. This is where the *marginal cost* changes from decreasing to increasing and so has its minimum value.

25. (a) $S = \dfrac{dR}{dx} = \dfrac{(1+4x^{0.4})(9.6x^{-0.6}) - (40+24x^{0.4})(1.6x^{-0.6})}{(1+4x^{0.4})^2}$

$$= \frac{9.6x^{-0.6} + 38.4x^{-0.2} - 64x^{-0.6} - 38.4x^{-0.2}}{(1+4x^{0.4})^2} = -\frac{54.4x^{-0.6}}{(1+4x^{0.4})^2}$$

(b)



At low levels of brightness, R is quite large $[R(0) = 40]$ and is quickly decreasing, that is, S is negative with large absolute value. This is to be expected: at low levels of brightness, the eye is more sensitive to slight changes than it is at higher levels of brightness.