Name: _____ ID#: _____

Midterm I

Math 1a
Introduction to Calculus

March 7, 2005

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

> *Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.*
>
> —*Handbook for Students*

Problem Number	Possible Points	Points Earned
1	20	
2	8	
3	10	
4	10	
5	10	
Total	58	

1 1

1. (20 Points) Let

$$f(x) = \begin{cases} -\dfrac{1}{x} & \text{if } x < 0; \\ x^2 - x & \text{if } 0 \leq x \leq 1; \\ x - 1 & \text{if } 1 < x < 2; \\ 0 & \text{if } x \geq 2. \end{cases}.$$

(a) Find the left- and right-hand limits at 0, 1, and 2. Justify your answers.

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(b) At what points is f not continuous? Justify.

(c) Does the graph of f have any asymptotes? Justify.

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2. (8 Points) Here is a table the percentage of low-weight (< 2500 grams) births among African-American mothers in various years.

Year	% low birthweight
1990	13.25
1991	13.55
1992	13.31
1993	13.34
1994	13.24
1995	13.13
1996	13.01
1997	13.01
1998	13.05
1999	13.11
2000	12.99
2001	12.95
2002	13.29

(i) What is the average rate of change in the low-birthweight percentage rate over the years 1992 to 1998?

(ii) Estimate the (instantenous) rate of change in the low-birthweight percentage rate in 1994.

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3. (10 Points)

(i) Let f be a function. State the *definition* of the derivative of f at a.

(ii) Let $f(x) = \dfrac{1}{x-2}$. Use the definition of the derivative to find $f'(a)$.

Note. No credit will be given for answers which use the product rule, quotient rule, power rule, *et cetera*.

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4. (10 Points) Find the following derivatives. Use any method you like.

(i) $\dfrac{d}{dx} 4x^3 + 3x^2 + 2x + 1$

(ii) $\dfrac{d}{dx} e^x(x-2)$.

(iii) $f'(x)$, where $f(x) = \dfrac{2x+3}{4x-2}$. *Simplify your answer.*

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5. (10 Points)

(i) State the Intermediate Value Theorem.

(ii) Use it to prove that there exists a number c between 0 and 1 which solves the equation

$$\arctan x = 2x - \frac{\pi}{3}.$$

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