Mathematics 1a, Section 2.6 Solutions

Alexander Ellis

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1. a. This is just the slope of the line through the two points

$$m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(3)}{x - 3}$$

b. This is the limit of the slope of the secant line PQ as Q approaches P

$$m = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$$

2. a. Average velocity is

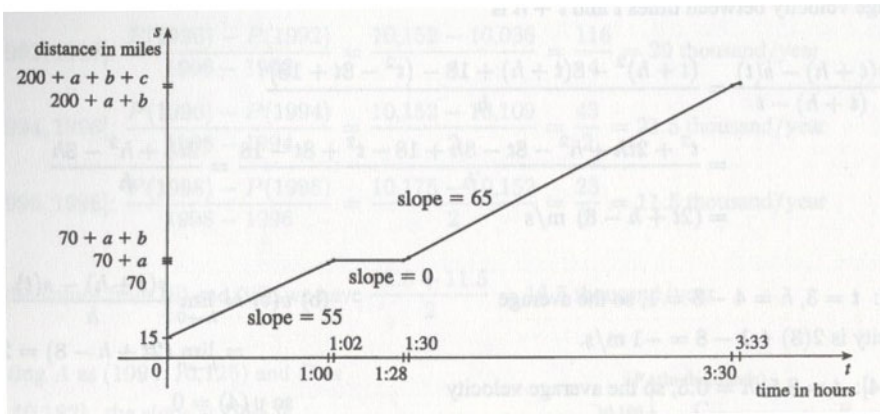
$$\frac{\Delta s}{\Delta t} = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}$$

b. Instantaneous velocity is

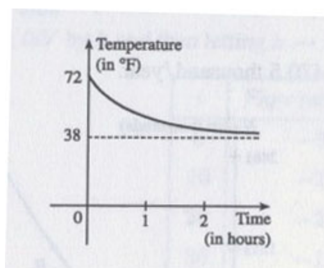
$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

3. The slope at D is the largest positive slope, followed by the positive slope at E. The slope at C is zero. The slope at B is steeper than at A (both are negative). In decreasing order, we have the slopes at: D, E, C, A, B.

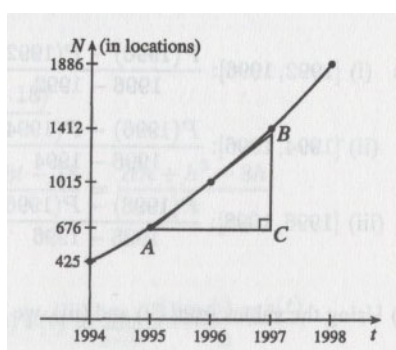
14. Let a denote the distance traveled from 1:00 to 1:02, b from 1:28 to 1:30, and c from 3:30 to 3:33, where all the times are relative to $t = 0$ at the beginning of the trip.



19. The sketch shows the graph for a room temperature of 72° and a refrigerator temperature of 38°. The initial rate of change is greater in magnitude the the rate of change after an hour.



24. a. i. [1995,1997]: $\frac{N(1997)-N(1995)}{1997-1995} = \frac{2461-873}{2} = 794$ thousand per year.
ii. [1995,1996]: $\frac{N(1996)-N(1995)}{1996-1995} = \frac{1513-873}{2} = 640$ thousand per year.
iii. [1994,1995]: $\frac{N(1995)-N(1994)}{1995-1994} = \frac{873-572}{1} = 301$ thousand per year.
b. Using the values from **a.ii** and **a.iii** we have $\frac{640+301}{2} = 470.5$ thousand per year.
c. Estimate A as $(1994, 420)$ and B as $(1996, 1275)$ the slope at 1995 is $\frac{1275-420}{1996-1994} = 427.5$ thousand per year.



26.

$$\begin{aligned}\Delta V = V(t+h) - V(t) &= 100,000\left(1-\frac{t+h}{60}\right)^2 - 100,000\left(1-\frac{t}{60}\right)^2 \\ &= 100,000\left[\left(1-\frac{t+h}{30}+\frac{(t+h)^2}{3600}\right)-\left(1-\frac{t}{30}+\frac{t^2}{3600}\right)\right] \\ &= 100,000\left(-\frac{h}{30}+\frac{2th}{3600}+\frac{h^2}{3600}\right) \\ &= \frac{250}{9}h(-120+2t+h)\end{aligned}$$

Dividing ΔV by h and then letting $h \to 0$, we see that the instantaneous rate of change is $\frac{500}{9}(t-60)$ gallons per minute.

t	Flow rate (gal/min)	Water remaining $V(t)$ (gal)
0	$-3333\bar{3}$	100,000
10	$-2777.\bar{7}$	$69,444.\bar{4}$
20	$-2222.\bar{2}$	$44,444.\bar{4}$
30	$-1666.\bar{6}$	25,000
40	$-1111.\bar{1}$	$11,111.\bar{1}$
50	$-555.\bar{5}$	$2,777.\bar{7}$
60	0	0

The magnitude of the flow rate is greatest at the beginning and gradually decreases to 0.