Mathematics 1a, Section 2.4 Solutions

Alexander Ellis

October 10, 2004

1. From Equation 1,

$$\lim_{x\to 4} f(x) = f(4)$$

4. g is continuous on $[-4,-2),(-2,2),[2,4),(4,6),(6,8)$.

6. See the picture for this problem.

8. **a.** Continuous; at the location in question, the temperature changes smoothly as time passes, without any instantaneous jumps from one temperature to another.

b. Continuous; the temperature at a specific time changes smoothly as the distance due west from New York City increases, without any instantaneous jumps.

c. Discontinuous; as the distance due west from New York City increases, the altitude above sea level may jump from one height to another without going through all of the intermediate values - at a cliff, for example.

d. Discontinuous; as the distance travelled increases, the cost of the ride jumps in small increments.

e. Discontinuous; when the lights are switched on (or off), the current suddenly changes between 0 and some nonzero value, without passing through all of the intermediate values. This is debatable, though, depending on your definition of current.

12. For $-4 < a < 4$, we have

$$\lim_{x\to a} f(x) = \lim_{x\to a} x\sqrt{16-x^2} = \lim_{x\to a} x\sqrt{\lim_{x\to a} 16 - \lim_{x\to a} x^2} = a\sqrt{16-a^2} = f(a)$$

so f is continuous on $(-4,4)$. Similarly, we get

$$\lim_{x\to 4^-} f(x) = 0 = f(4)$$
$$\lim_{x\to -4^+} f(x) = 0 = f(-4)$$

so f is continuous from the left at 4 and from the right at -4. Thus, f is continuous on $[-4,4]$.

16. See the picture for this problem.

$$f(x) = \begin{cases} 1+x^2 & x<1 \\ 4-x & x\geq 1 \end{cases} \quad \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(1+x^2) = 1+1^2 = 2$$
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(4-x) = 4-1 = 3$$

Thus, f is discontinuous at 1 because $\lim_{x\to 1} f(x)$ does not exist.

28. Because arctan is a continuous function, we can apply Theorem 8.

$$\lim_{x\to 2} \arctan\left(\frac{x^2-4}{3x^2-6x}\right) = \arctan\left(\lim_{x\to 2}\frac{(x+2)(x-2)}{3x(x-2)}\right) = \arctan\left(\lim_{x\to 2}\frac{x+2}{3x}\right) = \arctan\frac{2}{3} \approx 0.588$$

32. See the picture for this problem.

40. **a.** $f(x) = x^5 - x^2 + 2x + 3$ is continuous on $[-1,0]$, $f(-1) = -1 < 0$, and $f(0) = 3 > 0$. Since $-1 < 0 < 3$, there is a number c in $(-1,0)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $x^5 - x^2 + 2x + 3$ in the interval $(-1,0)$.

b. $f(-0.88) \approx -0.062 < 0$ and $f(-0.87) \approx 0.0047 > 0$, so there is a root between -0.88 and -0.87.