Mathematics 1a, Section 2.10 Solutions

Alexander Ellis

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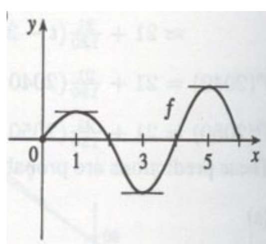
2. a. $f'(x) > 0$ and f is increasing on $(0, 1)$ and $(3, 5)$. $f'(x) < 0$ and f is decreasing on $(1, 3)$ and $(5, 6)$.

b. Since $f'(x) = 0$ at $x = 1$ and $x = 5$ and f' changes from positive to negative at both values, f changes from increasing to decreasing and has local maxima at $x = 1$ and $x = 5$. Since $f'(x) = 0$ at $x = 3$ and f' changes from negative to positive there, f changes from decreasing to increasing and has a local minimum at $x = 3$.

c.

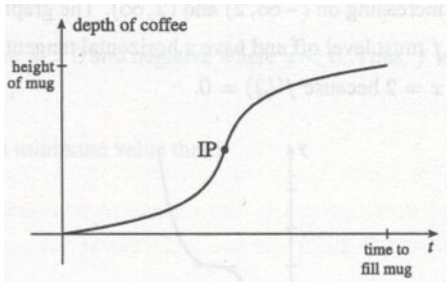


5. If $D(t)$ is the size of the deficit as a function of time, then at the time of the speech $D'(t) > 0$, but $D''(t) < 0$ because $D''(t) = (D')'(t)$ is the rate of change of $D'(t)$.

8. a. If the position function is increasing then the particle is moving toward the right.. This occurs on t-intervals $(0, 2)$ and $(4, 6)$. If the function is decreasing, then the particle is moving toward the left - that is, on $(2, 4)$.

b. The acceleration is the second derivative and is positive where the curve is concave upward. This occurs on $(3, 6)$. The acceleration is negative where the curve is concave downward - that is, on $(0, 3)$.

10. At first the depth increases slowly because the base of the mug is wide. But as the mug narrows, the coffee rises more quickly. Thus, the depth d increases at a increasing rate and its graph is concave upward. The rate of increase of d has a maximum where the mug is narrowest; that is, when the mug is half full. It is there that the inflection point (IP) occurs. Then the rate of increase of d starts to decrease as the mug widens and the graph becomes concave down.



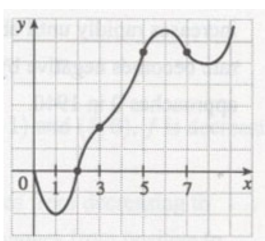
12. a. f is increasing where f' is positive, on $(1, 6)$ and $(8, \infty)$, and decreasing where f' is negative, on $(0, 1)$ and $(6, 8)$.

b. f has a local maximum where f' changes from positive to negative, at $x = 6$, and local minima where f' changes from negative to positive, at $x = 1$ and at $x = 8$.

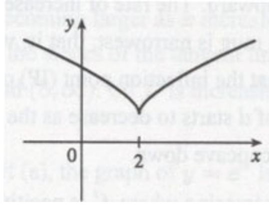
c. f is concave upward where f' is increasing, that is, on $(0, 2), (3, 5)$, and $(7, \infty)$, and concave downward where f' is decreasing, that is, on $(2, 3)$ and $(5, 7)$.

d. There are points of inflection where f' changes its direction of concavity, at $x = 2$, $x = 3$, $x = 5$, and $x = 7$.

e.



16. $f''(x) < 0$ on $(-\infty, 2)$ and $(2, \infty)$, so the graph of f is concave down on these intervals. Since f is not differentiable at 2, f could be discontinuous at 2 or have a cusp or corner at 2.



24. a.

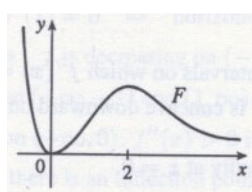
$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^4-2(x+h)^2-(x^4-2x^2)}{h}\\
&= \lim_{h\to 0}\frac{(x^4+4x^3h+6x^2h^2+4xh^3+h^4-2x^2-4xh-2h^2)-(x^4-2x^2)}{h}\\
&= \lim_{h\to 0}\left(4x^3+6x^2h+4xh^2+h^3-4x-2h\right) = 4x^3-4x\\
f''(x) &= \lim_{h\to 0}\frac{[4(x+h)^3-4(x+h)]-(4x^3-4x)}{h}\\
&= \lim_{h\to 0}\frac{(4x^3+12x^2h+12xh^2+4h^3-4x-4h)-(4x^3-4x)}{h}\\
&= \lim_{h\to 0}\frac{12x^2h+12xh^2+4h^3-4h}{h}\\
&= \lim_{h\to 0}(12x^2+12xh+4h^2-4) = 12x^2-4
\end{aligned}$$

b. $f'(x)>0 \Rightarrow 4x^3-4x>0 \Rightarrow 4x(x^2-1)>0 \Rightarrow 4x(x+1)(x-1)>0$, so f is increasing on $(-1,0)$ and $(1,\infty)$ and f is decreasing on $(-\infty,-1)$ and $(0,1)$.

c. $f''(x)>0 \Rightarrow 12x^2-4>0 \Rightarrow x^2>\frac{1}{3} \Rightarrow |x|>\sqrt{\frac{1}{3}}$, so f is concave up on $\left(-\infty,-\sqrt{\frac{1}{3}}\right)$ and $\left(\sqrt{\frac{1}{3}},\infty\right)$ and f is concave down on $\left(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}}\right)$.

26. We know right away that c cannot be f's antiderivative, since the slope of c is not zero at the x-value where $f=0$. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f.

27. The graph of F will have a minimum at 0 and a maximum at 2, since $f=F'$ goes from negative to positive at $x=0$, and from positive to negative at $x=2$.



28. The position function is the antiderivative of the velocity function, so its graph will have to be horizontal where the velocity function is equal to 0.