Name: _____ ID#: _____

Midterm II

Math 1a
Introduction to Calculus

8 December 2004

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

Please check your section:

☐	1.0	MWF10	Tatyana Chmutova	☐	4.0	TΘ10	Dawei Chen
☐	1.1	MWF10	Matthew Leingang	☐	4.1	TΘ10	Jerrel Mast
☐	2.0	MWF11	Ethan Cotterill	☐	4.2	TΘ10	Chun-Chun Wu
☐	3.0	MWF12	Matt Bainbridge	☐	5.0	TΘ11:30	Derek Bruff
				☐	5.1	TΘ11:30	Sonal Jain

Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.

—Handbook for Students

Problem Number	Possible Points	Points Earned
1	12	
2	10	
3	10	
4	10	
5	20	
6	12	
7	6	
8	10	
9	10	
Total	100	

1 1

1. (12 Points) Find the following derivatives. You may use any "standard" facts about differentiation.

(i) $\dfrac{d}{du}(u^3+2)^{17}$

(ii) $\dfrac{d}{dx}\left(e^{\sin(x)+19}\right)$

(iii) $\dfrac{d}{ds}\ln(\tan s)$

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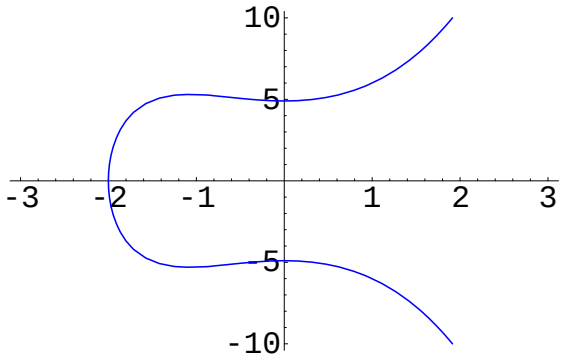
2. (10 Points) Consider the equation

$$y^2 = (x^2 + 3)(x^3 + 8)$$

This equation defines a curve in the plane.



(a) Calculate $\frac{dy}{dx}$.

2 **2**

(b) The graph of the curve has two tangent lines at $x = 1$. Find the point at which these two lines intersect.

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3 3

3. (10 Points) In this question, you will use logarithmic differentiation to find the derivative of a product of functions without using the product rule. Assume that $f(x)$ and $g(x)$ are always positive.

(a) Take the natural log of both sides of the equation $y = f(x)g(x)$ and use properties of logarithms to rewrite the right-hand side.

(b) Differentiate your answer to part (a) with respect to x.

3 3

(c) Use algebra to write $\frac{dy}{dx}$ as a function of x only. (y cannot appear in your formula for $\frac{dy}{dx}$.)

(d) Is this the same answer you would have obtained by applying the product rule directly?

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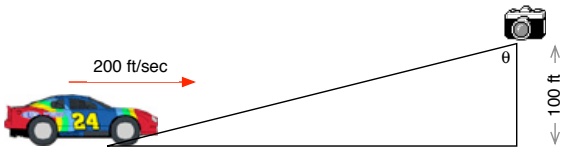
4. (10 Points) You are videotaping a race from a stand 100 feet from the track, following a car that is moving 200 feet per second. How fast will your camera angle θ be changing two seconds before the car reaches the point on the track that is directly in front of you? Assume the track is a straight line.



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5. (20 Points) Let $f(x) = \dfrac{1}{x-1} + \dfrac{1}{2(x-1)^2}$.

(a) State the domain of f.

(b) Find zeroes of f and the intervals where it is positive or negative.

(c) Find all vertical and horizontal asymptotes of f.

5 **5**

(d) Find the intervals of increase or decrease, and any local maxima or minima.

(e) Find the intervals of concavity and the inflection points.

5 **5**

(f) Sketch the graph of f. Find the global minimum and maximum, if they exist.

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6 6

6. (12 Points) Find each of the following limits, with justification. Indicate each point at which you apply L'Hôpital's rule.

(i) $\lim_{x\to 0} \dfrac{\arcsin(x)}{\ln(x+1)}$

Reminder. $\arcsin(x)$ is written in the book as $\sin^{-1}(x)$, but this is *not* the same thing as $\dfrac{1}{\sin(x)}$.

(ii) $\lim_{x\to 0} \dfrac{\sin(x)}{x+1}$

(iii) $\lim_{x\to 0^+} \sin(x)^{\sin(x)}$

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7. (6 Points) Use linear approximation to find the approximate value of $\ln(1.02)$.

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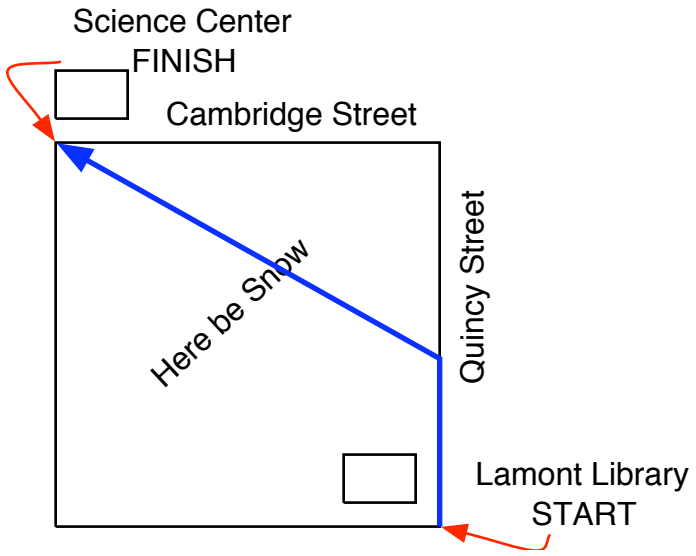
8. (10 Points) Ferdbert Freshman wants to get from his study carrell at Lamont Library to a problem session at the Science Center. The most direct path would be diagonally across Harvard Yard. Unfortunately, a foot of snow has just fallen, and Ferdbert walks only $\frac{1}{5}$ as fast on snow as on pavement. The sidewalks around the Yard (along Quincy Street and Cambridge Street) are plowed. Ferdbert decides to take a mixed approach, walking North along Quincy Street for a certain distance, then cutting diagonally across the rest of the yard to the Science Center. What is the minimum amount of time that Ferdbert can take to make his trip?



We'll assume that Harvard Yard is a square of side length 200 meters, and Ferdbert's normal walking speed is 10 kilometers per hour. Also, ignore the fact that there is a wall around Harvard Yard and there are only finitely many places to physically enter.

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9. (10 Points) Sharks and snorks used to coexist peacefully in a far away ocean; at one point in time there were 100 snorks and they reproduced at the rate of 4 new snorks every day. A year passed, however, and the sharks could no longer repress the urge to sink their teeth into those juicy snorks. Although the snorks kept reproducing at their normal rate, sharks began eating snorks at a rate of *at most* 2.5 snorks eaten every day, and *at least* 2.3 snorks eaten every day.

Assume that the snork population is a differentiable function of time. Give upper and lower estimates for the population of snorks t days after the onset of the shark attacks. Use the Mean Value Theorem to justify your assertions.

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