Solutions #5
Math 1a

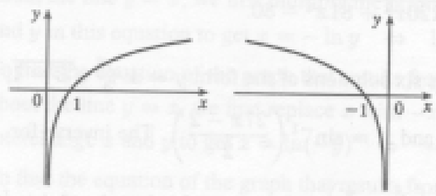
1.6: 4, 22, 48, 58

4. f is one-to-one since for any two different domain values, there are different range values.

22. $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}} \Rightarrow 1 - \dfrac{v^2}{c^2} = \dfrac{m_0^2}{m^2} \Rightarrow \dfrac{v^2}{c^2} = 1 - \dfrac{m_0^2}{m^2} \Rightarrow v^2 = c^2\left(1 - \dfrac{m_0^2}{m^2}\right) \Rightarrow$
$v = c\sqrt{1 - \dfrac{m_0^2}{m^2}}$. This formula gives us the velocity v of the particle in terms of its mass m, that is, $v = f^{-1}(m)$.

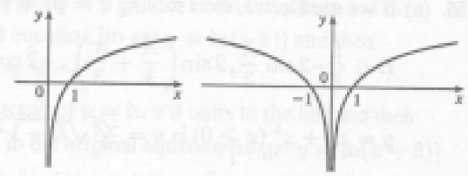
48. (a) Reflect the graph of $y = \ln x$ about the y-axis to obtain the graph of $y = \ln(-x)$.

$y = \ln x$ $\qquad$ $y = \ln(-x)$



(b) Reflect the portion of the graph of $y = \ln x$ to the right of the y-axis about the y-axis. The graph of $y = \ln|x|$ is that reflection in addition to the original portion.

$y = \ln x$ $\qquad$ $y = \ln|x|$

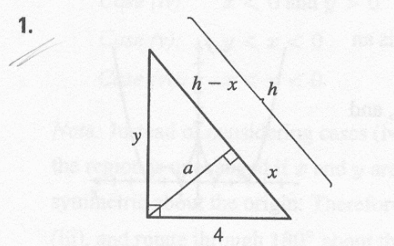


58. (a) $Q = Q_0\left(1 - e^{-t/a}\right) \Rightarrow \dfrac{Q}{Q_0} = 1 - e^{-t/a} \Rightarrow e^{-t/a} = 1 - \dfrac{Q}{Q_0} \Rightarrow -\dfrac{t}{a} = \ln\left(1 - \dfrac{Q}{Q_0}\right) \Rightarrow$
$t = -a\ln(1 - Q/Q_0)$. This gives us the time t necessary to obtain a given charge Q.

(b) $Q = 0.9Q_0$ and $a = 2 \Rightarrow t = -2\ln(1 - 0.9(Q_0/Q_0)) = -2\ln 0.1 \approx 4.6$ seconds.

p.93: 1, 3, 12, 17

1.



By using the area formula for a triangle, $\frac{1}{2}$ (base) (height), in two ways, we see that $\frac{1}{2}(4)(y) = \frac{1}{2}(h)(a)$, so $a = \dfrac{4y}{h}$. Since $4^2 + y^2 = h^2$, $y = \sqrt{h^2 - 16}$, and $a = \dfrac{4\sqrt{h^2 - 16}}{h}$.

Solutions #5
Math 1a

3. $|2x-1| = \begin{cases} 1-2x & \text{if } x < \frac{1}{2} \\ 2x-1 & \text{if } x \geq \frac{1}{2} \end{cases}$ and $|x+5| = \begin{cases} -x-5 & \text{if } x < -5 \\ x+5 & \text{if } x \geq -5 \end{cases}$

Therefore, we consider the three cases $x < -5$, $-5 \leq x < \frac{1}{2}$, and $x \geq \frac{1}{2}$.

If $x < -5$, we must have $1 - 2x - (-x-5) = 3 \Leftrightarrow x = 3$, which is false, since we are considering $x < -5$.

If $-5 \leq x < \frac{1}{2}$, we must have $1 - 2x - (x+5) = 3 \Leftrightarrow x = -\frac{7}{3}$.

If $x \geq \frac{1}{2}$, we must have $2x - 1 - (x+5) = 3 \Leftrightarrow x = 9$.

So the two solutions of the equation are $x = -\frac{7}{3}$ and $x = 9$.

12. (a) $f(-x) = \ln\left(-x + \sqrt{(-x)^2+1}\right) = \ln\left(-x + \sqrt{x^2+1} \cdot \frac{-x-\sqrt{x^2+1}}{-x-\sqrt{x^2+1}}\right)$

$$= \ln\left(\frac{x^2-(x^2+1)}{-x-\sqrt{x^2+1}}\right) = \ln\left(\frac{-1}{-x-\sqrt{x^2+1}}\right) = \ln\left(\frac{1}{x+\sqrt{x^2+1}}\right)$$

$$= \ln 1 - \ln\left(x+\sqrt{x^2+1}\right) = -\ln\left(x+\sqrt{x^2-1}\right) = -f(x)$$

(b) $y = \ln\left(x+\sqrt{x^2+1}\right)$. Interchanging x and y, we get $x = \ln\left(y+\sqrt{y^2+1}\right) \Rightarrow e^x = y + \sqrt{y^2+1} \Rightarrow$

$e^x - y = \sqrt{y^2+1} \Rightarrow e^{2x} - 2ye^x + y^2 = y^2 + 1 \Rightarrow e^{2x} - 1 = 2ye^x \Rightarrow$

$y = \dfrac{e^{2x}-1}{2e^x} = f^{-1}(-x)$

17. Let S_n be the statement that $7^n - 1$ is divisible by 6.

- S_1 is true because $7^1 - 1 = 6$ is divisible by 6.
- Assume S_k is true, that is, $7^k - 1$ is divisible by 6. In other words, $7^k - 1 = 6m$ for some positive integer m. Then $7^{k+1} - 1 = 7^k \cdot 7 - 1 = (6m+1) \cdot 7 - 1 = 42m + 6 = 6(7m+1)$, which is divisible by 6, so S_{k+1} is true.
- Therefore, by mathematical induction, $7^n - 1$ is divisible by 6 for every positive integer n.