Math 1a
Solution Set #1
Page 1

2. (a) The point $(-4, -2)$ is on the graph of f, so $f(-4) = -2$. The point $(3, 4)$ is on the graph of g, so $g(3) = 4$.

(b) We are looking for the values of x for which the y-values are equal. The y-values for f and g are equal at the points $(-2, 1)$ and $(2, 2)$, so the desired values of x are -2 and 2.

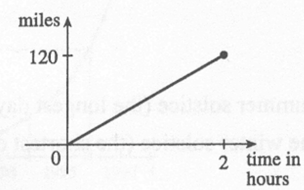
(c) $f(x) = -1$ is equivalent to $y = -1$. When $y = -1$, we have $x = -3$ and $x = 4$.

(d) As x increases from 0 to 4, y decreases from 3 to -1. Thus, f is decreasing on the interval $[0, 4]$.

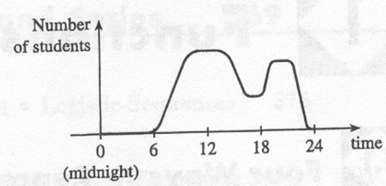
(e) The domain of f consists of all x-values on the graph of f. For this function, the domain is $-4 \le x \le 4$, or $[-4, 4]$. The range of f consists of all y-values on the graph of f. For this function, the range is $-2 \le y \le 3$, or $[-2, 3]$.

(f) The domain is $[-4, 3]$ and the range is $[0.5, 4]$.

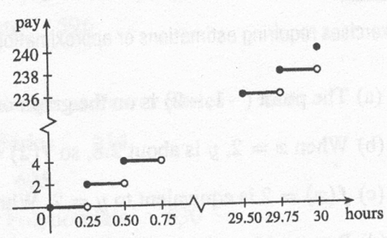
4. *Example 1:* A car is driven at 60 mi/h for 2 hours. The distance d traveled by the car is a function of the time t. The domain of the function is $\{t \mid 0 \le t \le 2\}$, where t is measured in hours. The range of the function is $\{d \mid 0 \le d \le 120\}$, where d is measured in miles.



Example 2: At a certain university, the number of students N on campus at any time on a particular day is a function of the time t after midnight. The domain of the function is $\{t \mid 0 \le t \le 24\}$, where t is measured in hours. The range of the function is $\{N \mid 0 \le N \le k\}$, where N is an integer and k is the largest number of students on campus at once.



Example 3: A certain employee is paid \$8.00 per hour and works a maximum of 30 hours per week. The number of hours worked is rounded down to the nearest quarter of an hour. This employee's gross weekly pay P is a function of the number of hours worked h. The domain of the function is $\{0, 0.25, 0.5, \ldots, 29.75, 30\}$ and the range of the function is $\{0, 2.00, 4.00, \ldots, 238.00, 240.00\}$.



8. Yes, the curve is the graph of a function with domain $[-3, 2]$ and range $\{-2\} \cup (0, 3]$.

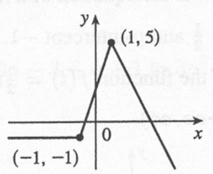
10. The salesman travels away from home from 8 to 9 A.M. and is then stationary until 10:00. The salesman travels farther away from 10 until noon. There is no change in his distance from home until 1:00, at which time the distance from home decreases until 3:00. Then the distance starts increasing again, reaching the maximum distance away from home at 5:00. There is no change from 5 until 6, and then the distance decreases rapidly until 7:00 P.M., at which time the salesman reaches home.

24. $f(x) = (5x+4)\,/\,(x^2+3x+2)$ is defined for all x except when $0 = x^2+3x+2 \quad \Leftrightarrow$
$0 = (x+2)(x+1) \quad \Leftrightarrow \quad x = -2$ or -1, so the domain is
$\{x \in \mathbb{R} \,|\, x \neq -2, -1\} = (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

26. $g(u) = \sqrt{u} + \sqrt{4-u}$ is defined when $u \geq 0$ and $4 - u \geq 0 \quad \Leftrightarrow \quad u \leq 4$. Thus, the domain is $0 \leq u \leq 4 = [0, 4]$.

36. $f(x) = \begin{cases} -1 & \text{if } x \leq -1 \\ 3x+2 & \text{if } -1 < x < 1 \\ 7-2x & \text{if } x \geq 1 \end{cases}$

Domain is $\mathbb{R}$.



48. The area of the window is $A = xh + \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = xh + \dfrac{\pi x^2}{8}$, where h is the height of the rectangular portion of the window. The perimeter is $P = 2h + x + \frac{1}{2}\pi x = 30 \quad \Leftrightarrow \quad 2h = 30 - x - \frac{1}{2}\pi x \quad \Leftrightarrow \quad h = \frac{1}{4}(60 - 2x - \pi x)$.
Thus,

$$A(x) = x\,\frac{60 - 2x - \pi x}{4} + \frac{\pi x^2}{8} = 15x - \tfrac{1}{2}x^2 - \tfrac{\pi}{4}x^2 + \tfrac{\pi}{8}x^2 = 15x - \tfrac{4}{8}x^2 - \tfrac{\pi}{8}x^2 = 15x - x^2\left(\tfrac{\pi+4}{8}\right)$$

Since the lengths x and h must be positive quantities, we have $x > 0$ and $h > 0$. For $h > 0$, we have $2h > 0 \quad \Leftrightarrow$
$30 - x - \frac{1}{2}\pi x > 0 \quad \Leftrightarrow \quad 60 > 2x + \pi x \quad \Leftrightarrow \quad x < \dfrac{60}{2+\pi}$. Hence, the domain of A is $0 < x < \dfrac{60}{2+\pi}$.