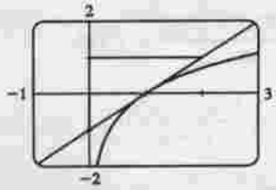
2.9

2. (a)

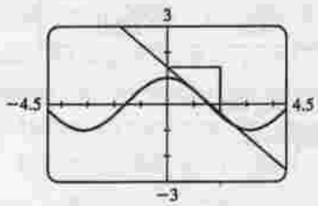


Consider the triangle in the graph. The vertices that lie on the tangent line are $(0, -1)$ and $(2, 1)$. Check these points with the linearization in part (b). Thus, $f'(1) \approx \Delta y/\Delta x = [1-(-1)]/(2-0) = 1$.

(b) Using $f(x) = \ln x$ and $a = 1$, we have $L(x) = f(1) + f'(1)(x-1) = 0 + 1(x-1) = x - 1$.
$L(0.9) = 0.9 - 1 = -0.1$ and $L(1.3) = 0.3$. So we estimate that $\ln 0.9 \approx -0.1$ and $\ln 1.3 \approx 0.3$.

(c) From the graph, we see that the tangent line lies *above* the curve, so the estimates are *greater* than the values.

4. (a)



From the triangle in the graph, $\dfrac{\Delta y}{\Delta x} \approx \dfrac{-0.3 - 1.4}{2 - 0} = -0.85$.

(b) $L(x) = f(\frac{\pi}{3}) + f'(\frac{\pi}{3})(x - \frac{\pi}{3}) \approx \frac{1}{2} - 0.85(x - \frac{\pi}{3}) \approx -0.85x + 1.39$.
Remember that this estimate is dependent on the estimates made in part (a).

(c) $\cos 1 \approx L(1) = 0.54$, $\cos 1.1 \approx L(1.1) = 0.455$, $\cos 1.5 \approx L(1.5) = 0.115$, and $\cos 2 \approx L(2) = -0.31$.
Compare those values with the following calculator values: $\cos 1 \approx 0.5403$, $\cos 1.1 \approx 0.4536$, $\cos 1.5 \approx 0.0707$, and $\cos 2 \approx -0.4161$. [It turns out that $L(1) < f(1)$, but only because of the "eyeball" estimates in part (a).] The estimates that are most accurate are those at $x = 1$ and $x = 1.1$, since they are closest to $x = \frac{\pi}{3} \approx 1.047$.

(d) The graph in part (a) shows that the tangent line lies above the curve for $1 \leq x \leq 2$. That explains why the estimates are overestimate

6. (a) From Exercise 2.8.18(e) for $f(x) = x^3$, $f'(x) = 3x^2$, so $f'(1) = 3$.

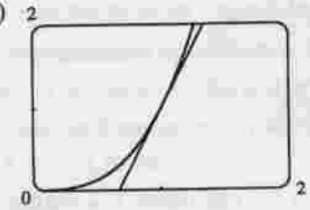
(b) $L(x) = f(1) + f'(1)(x - 1)$

$= 1 + 3(x - 1) = 3x - 2$

x	$L(x) = 3x - 2$	$f(x) = x^3$
0.9	0.7	0.729
0.95	0.85	0.857375
0.99	0.97	0.970299
1.01	1.03	1.030301
1.05	1.15	1.157625
1.1	1.3	1.331

The estimates using L are all underestimates of the actual function values.

(c)

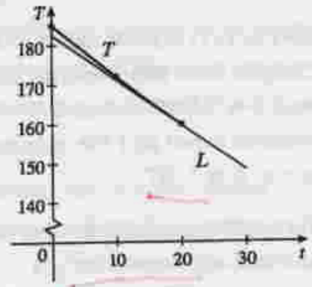


Since the tangent line lies under the graph, our underestimate claim in part (b) is supported.

7. As in Example 3, $T(0) = 185$, $T(10) = 172$, $T(20) = 160$, and

$T'(20) \approx \dfrac{T(10) - T(20)}{10 - 20} = \dfrac{172 - 160}{-10} = -1.2\ °\text{F/min}$.

$T(30) \approx T(20) + T'(20)(30 - 20) \approx 160 - 1.2(10) = 148\ °\text{F}$.



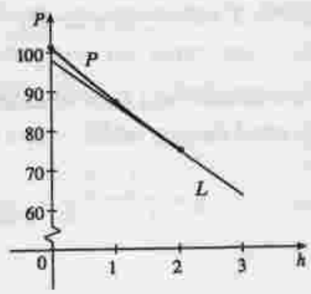
We would expect the temperature of the turkey to get closer to 75 °F as time increases. Since the temperature decreased 13 °F in the first 10 minutes and 12 °F in the second 10 minutes, we can assume that the slopes of the tangent line are increasing through negative values: $-1.3, -1.2, \ldots$. Hence, the tangent lines are under the curve and 148 °F is an underestimate. From the figure, we estimate the slope of the tangent line at $t = 20$ to be $\frac{184 - 147}{0 - 30} = -\frac{37}{30}$. Then the linear approximation becomes $T(30) \approx T(20) + T'(20) \cdot 10 \approx 160 - \frac{37}{30}(10) = 147\frac{2}{3} \approx 147.7$.

8. $P'(2) \approx \dfrac{P(1) - P(2)}{1 - 2} = \dfrac{87.1 - 74.9}{-1} = -12.2$ kilopascals/km.

$P(3) \approx P(2) + P'(2)(3 - 2) \approx 74.9 - 12.2(1) = 62.7$ kPa.

From the figure, we estimate the slope of the tangent line at $h = 2$ to be $\frac{98 - 63}{0 - 3} = -\frac{35}{3}$. Then the linear approximation becomes

$P(3) \approx P(2) + P'(2) \cdot 1 \approx 74.9 - \frac{35}{3} \approx 63.23$ kPa.



10. Let $A = \dfrac{N(1980) - N(1985)}{1980 - 1985} = \dfrac{15.0 - 17.0}{-5} = 0.4$ and $B = \dfrac{N(1990) - N(1985)}{1990 - 1985} = \dfrac{19.3 - 17.0}{5} = 0.46$.

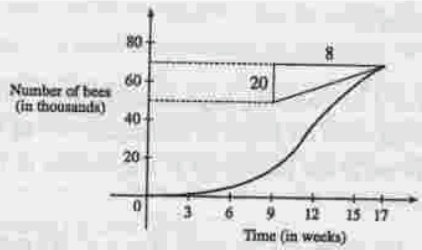
Then $N'(1985) = \lim\limits_{t \to 1985} \dfrac{N(t) - N(1985)}{t - 1985} \approx \dfrac{A + B}{2} = 0.43$ million/year. So

$N(1984) \approx N(1985) + N'(1985)(1984 - 1985) \approx 17.0 + 0.43(-1) = 16.57$ million.

$N'(2000) \approx \dfrac{N(1995) - N(2000)}{1995 - 2000} = \dfrac{22.0 - 24.9}{-5} = 0.58$ million/year.

$N(2006) \approx N(2000) + N'(2000)(2006 - 2000) \approx 24.9 + 0.58(6) = 28.38$ million.

12. (a)



From the figure,

$P'(17) \approx \frac{20}{8} = 2.5$ thousand bees/week.

$P(18) \approx P(17) + P'(17)(18 - 17)$

$\approx 70 + 2.5(1) = 72.5$ or 72,500 bees.

$P(20) \approx P(17) + P'(17)(20 - 17)$

$\approx 70 + 2.5(3) = 77.5$ or 77,500 bees.

(b) Since the tangent line at $t = 17$ is above the graph, our predictions are overestimates.

(c) $P(18)$ is more accurate than $P(20)$ since it is closer to the given data.

13. (a) The graph shows that $f'(1) = 2$, so $L(x) = f(1) + f'(1)(x - 1) = 5 + 2(x - 1) = 2x + 3$.
$f(0.9) \approx L(0.9) = 4.8$ and $f(1.1) \approx L(1.1) = 5.2$.

(b) From the graph, we see that $f'(x)$ is positive and decreasing. This means that the slopes of the tangent lines are positive, but the tangents are becoming less steep. So the tangent lines lie *above* the curve. Thus, the estimates in part (a) are too large.