3.1

3.1 Derivatives of Polynomials and Exponential Functions

1. (a) e is the number such that $\lim_{h\to 0}\frac{e^h-1}{h}=1$.

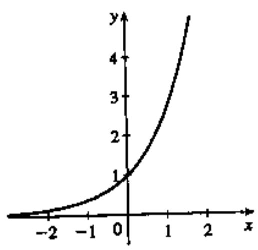
(b)

x	$(2.7^x-1)/x$
−0.001	0.9928
−0.0001	0.9932
0.001	0.9937
0.0001	0.9933

x	$(2.8^x-1)/x$
−0.001	1.0291
−0.0001	1.0296
0.001	1.0301
0.0001	1.0297

From the tables (to two decimal places), $\lim_{h\to 0}\frac{2.7^h-1}{h}=0.99$ and $\lim_{h\to 0}\frac{2.8^h-1}{h}=1.03$. Since $0.99<1<1.03$, $2.7<e<2.8$.

2. (a)



The function value at $x=0$ is 1 and the slope at $x=0$ is 1.

(b) $f(x)=e^x$ is an exponential function and $g(x)=x^e$ is a power function. $\frac{d}{dx}(e^x)=e^x$ and $\frac{d}{dx}(x^e)=ex^{e-1}$.

(c) $f(x)=e^x$ grows more rapidly than $g(x)=x^e$ when x is large.

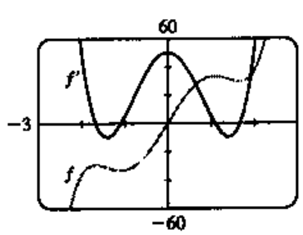
9. $G(x)=\sqrt{x}-2e^x=x^{1/2}-2e^x \quad\Rightarrow\quad G'(x)=\frac{1}{2}x^{-1/2}-2e^x=\frac{1}{2\sqrt{x}}-2e^x$

19. $v=t^2-\frac{1}{\sqrt[4]{t^3}}=t^2-t^{-3/4} \quad\Rightarrow\quad v'=2t-\left(-\frac{3}{4}\right)t^{-7/4}=2t+\frac{3}{4t^{7/4}}=2t+\frac{3}{4t\sqrt[4]{t^3}}$

22. $u=\sqrt[3]{t^2}+2\sqrt{t^3}=t^{2/3}+2t^{3/2} \quad\Rightarrow\quad u'=\frac{2}{3}t^{-1/3}+2\left(\frac{3}{2}\right)t^{1/2}=\frac{2}{3\sqrt[3]{t}}+3\sqrt{t}$

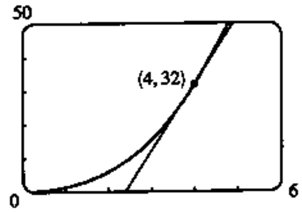
24. $f(x)=3x^5-20x^3+50x \quad\Rightarrow$

$f'(x)=15x^4-60x^2+50$.

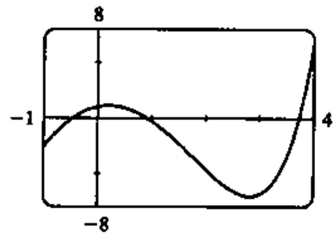


Notice that $f'(x)=0$ when f has a horizontal tangent and that f' is an even function while f is an odd function.

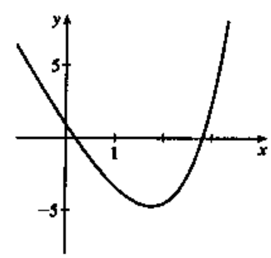
32. $y = f(x) = x^{5/2} \Rightarrow f'(x) = \frac{5}{2}x^{3/2}$. So the slope of the tangent line at $(4, 32)$ is $f'(4) = 20$ and its equation is $y - 32 = 20(x - 4)$ or $y = 20x - 48$.



36. (a)



(b)



From the graph in part (a), it appears that f' is zero at $x_1 \approx 0.2$ and $x_2 \approx 2.8$. The slopes are positive (so f' is positive) on $(-\infty, x_1)$ and (x_2, ∞). The slopes are negative (so f' is negative) on (x_1, x_2).

(c) $g(x) = e^x - 3x^2 \Rightarrow g'(x) = e^x - 6x$