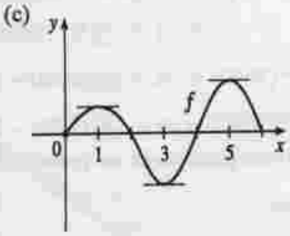
2.10

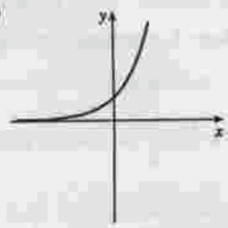
2. (a) $f'(x) > 0$ and f is increasing on $(0, 1)$ and $(3, 5)$. $f'(x) < 0$ and f is decreasing on $(1, 3)$ and $(5, 6)$.

(b) Since $f'(x) = 0$ at $x = 1$ and $x = 5$ and f' changes from positive to negative at both values, f changes from increasing to decreasing and has local maxima at $x = 1$ and $x = 5$. Since $f'(x) = 0$ at $x = 3$ and f' changes from negative to positive there, f changes from decreasing to increasing and has a local minimum at $x = 3$.

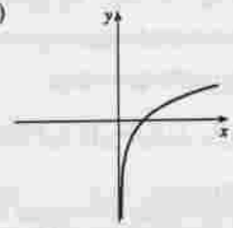
(c)



4. (a)



(b)



(c) In part (a), the graph of $y = e^x$ is a curve whose slope is always positive and increasing. In part (b), the graph of $y = \ln$ is a curve whose slope is always positive and decreasing.

5. If $D(t)$ is the size of the deficit as a function of time, then at the time of the speech $D'(t) > 0$, but $D''(t) < 0$ because $D''(t) = (D')'(t)$ is the rate of change of $D'(t)$.

6. (a) The rate of increase of the population is initially very small, then gets larger until it reaches a maximum at about $t = 8$ hours, and decreases toward 0 as the population begins to level off.

(b) The rate of increase has its maximum value at $t = 8$ hours.

(c) The population function is concave upward on $(0, 8)$ and concave downward on $(8, 18)$.

(d) At $t = 8$, the population is about 350, so the inflection point is about $(8, 350)$.

8. (a) If the position function is increasing, then the particle is moving toward the right. This occurs on t-intervals $(0, 2)$ and $(4, 6)$. If the function is decreasing, then the particle is moving toward the left—that is, on $(2, 4)$.

(b) The acceleration is the second derivative and is positive where the curve is concave upward. This occurs on $(3, 6)$. The acceleration is negative where the curve is concave downward—that is, on $(0, 3)$.

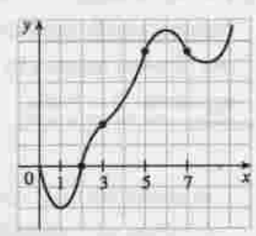
12. (a) f is increasing where f' is positive, on $(1, 6)$ and $(8, \infty)$, and decreasing where f' is negative, on $(0, 1)$ and $(6, 8)$.

(b) f has a local maximum where f' changes from positive to negative, at $x = 6$, and local minima where f' changes from negative to positive, at $x = 1$ and at $x = 8$.

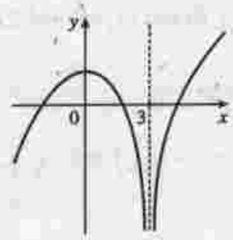
(c) f is concave upward where f' is increasing, that is, on $(0, 2)$, $(3, 5)$, and $(7, \infty)$, and concave downward where f' is decreasing, that is, on $(2, 3)$ and $(5, 7)$.

(d) There are points of inflection where f changes its direction of concavity, at $x = 2$, $x = 3$, $x = 5$ and $x = 7$.

(e)



20. $\lim_{x \to 3} f(x) = -\infty \quad \Rightarrow \quad$ there is a vertical asymptote at $x = 3$. $f'(0) = 0$ means that there is a horizontal tangent at $x = 0$. $f'(x) > 0$ if $x < 0$ or $x > 3$ and $f'(x) < 0$ if $0 < x < 3$ indicates that there is a local maximum at $x = 0$, since f is increasing on $(-\infty, 0)$ and decreasing on $(0, 3)$, and then increasing on $(3, \infty)$. $f''(x) < 0$ if $x \neq 3 \quad \Rightarrow \quad f$ is concave downward on $(-\infty, 3)$ and $(3, \infty)$.



27. The graph of F will have a minimum at 0 and a maximum at 2, since $f = F'$ goes from negative to positive at $x = 0$, and from positive to negative at $x = 2$.