2.6 Tangents, Velocities, and Other Rates of Change

1. (a) This is just the slope of the line through two points: $m_{PQ} = \dfrac{\Delta y}{\Delta x} = \dfrac{f(x) - f(3)}{x - 3}$.

(b) This is the limit of the slope of the secant line PQ as Q approaches P: $m = \lim\limits_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$.

2. (a) Average velocity $= \dfrac{\Delta s}{\Delta t} = \dfrac{f(a+h) - f(a)}{(a+h) - a} = \dfrac{f(a+h) - f(a)}{h}$

(b) Instantaneous velocity $= \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

3. The slope at D is the largest positive slope, followed by the positive slope at E. The slope at C is zero. The slope at B is steeper than at A (both are negative). In decreasing order, we have the slopes at: D, E, C, A, B.

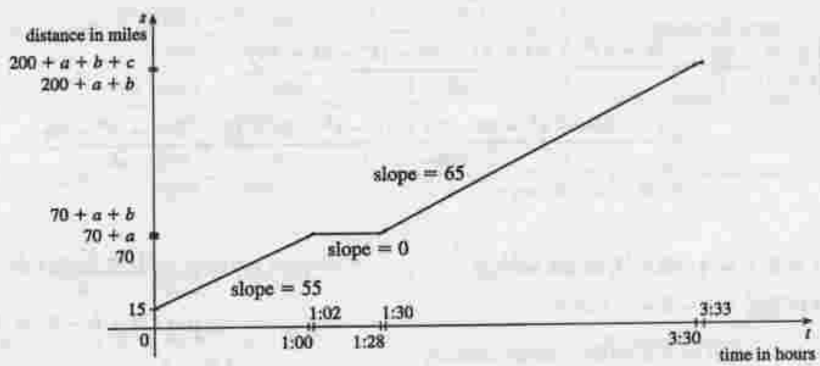
13. (a) Since the slope of the tangent at $t = 0$ is 0, the car's initial velocity was 0.

(b) The slope of the tangent is greater at C than at B, so the car was going faster at C.

(c) Near A, the tangent lines are becoming steeper as x increases, so the velocity was increasing, so the car was speeding up. Near B, the tangent lines are becoming less steep, so the car was slowing down. The steepest tangent near C is the one at C, so at C the car had just finished speeding up, and was about to start slowing down.

(d) Between D and E, the slope of the tangent is 0, so the car did not move during that time.

14. Let a denote the distance traveled from 1:00 to 1:02, b from 1:28 to 1:30, and c from 3:30 to 3:33, where all the times are relative to $t = 0$ at the beginning of the trip.



18. (a) The average velocity between times t and $t+h$ is

$$\frac{s(t+h)-s(t)}{(t+h)-t} = \frac{(t+h)^2 - 8(t+h) + 18 - (t^2 - 8t + 18)}{h}$$
$$= \frac{t^2 + 2th + h^2 - 8t - 8h + 18 - t^2 + 8t - 18}{h} = \frac{2th + h^2 - 8h}{h}$$
$$= (2t + h - 8)\ \text{m/s}$$

(i) $[3,4]$: $t = 3$, $h = 4 - 3 = 1$, so the average velocity is $2(3) + 1 - 8 = -1\ \text{m/s}$.

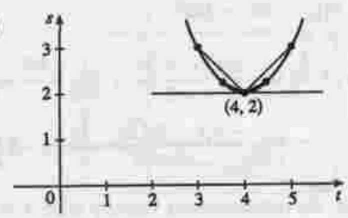
(ii) $[3.5, 4]$: $t = 3.5$, $h = 0.5$, so the average velocity is $2(3.5) + 0.5 - 8 = -0.5\ \text{m/s}$.

(iii) $[4, 5]$: $t = 4$, $h = 1$, so the average velocity is $2(4) + 1 - 8 = 1\ \text{m/s}$.

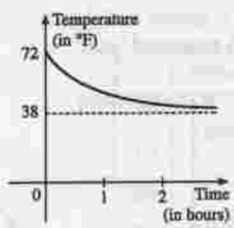
(iv) $[4, 4.5]$: $t = 4$, $h = 0.5$, so the average velocity is $2(4) + 0.5 - 8 = 0.5\ \text{m/s}$.

(b) $v(t) = \lim_{h\to 0} \frac{s(t+h)-s(t)}{h}$
$= \lim_{h\to 0} (2t + h - 8) = 2t - 8$,
so $v(4) = 0$.

(c)



19. The sketch shows the graph for a room temperature of 72° and a refrigerator temperature of 38°. The initial rate of change is greater in magnitude than the rate of change after an hour.



22. (a) (i) $[1992, 1996]$: $\dfrac{P(1996) - P(1992)}{1996 - 1992} = \dfrac{10{,}152 - 10{,}036}{4} = \dfrac{116}{4} = 29$ thousand/year

(ii) $[1994, 1996]$: $\dfrac{P(1996) - P(1994)}{1996 - 1994} = \dfrac{10{,}152 - 10{,}109}{2} = \dfrac{43}{2} = 21.5$ thousand/year

(iii) $[1996, 1998]$: $\dfrac{P(1998) - P(1996)}{1998 - 1996} = \dfrac{10{,}175 - 10{,}152}{2} = \dfrac{23}{2} = 11.5$ thousand/year

(b) Using the values from (ii) and (iii), we have $\dfrac{21.5 + 11.5}{2} = 16.5$ thousand/year.

(c) Estimating A as $(1994, 10{,}125)$ and B as $(1998, 10{,}182)$, the slope at 1996 is $\dfrac{10{,}182 - 10{,}125}{1998 - 1994} = \dfrac{57}{4} = 14.25$ thousand/year.