2.5

1. (a) As x approaches 2 (from the right or the left), the values of $f(x)$ become large.

(b) As x approaches 1 from the right, the values of $f(x)$ become large negative.

(c) As x becomes large, the values of $f(x)$ approach 5.

(d) As x becomes large negative, the values of $f(x)$ approach 3.

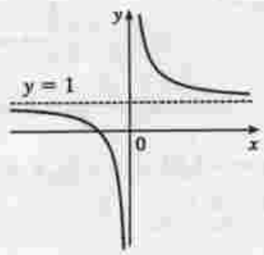
4. (a) $\lim_{x\to\infty} g(x) = 2$ (b) $\lim_{x\to-\infty} g(x) = -2$

(c) $\lim_{x\to3} g(x) = \infty$ (d) $\lim_{x\to0} g(x) = -\infty$

(e) $\lim_{x\to-2^+} g(x) = -\infty$ (f) Vertical: $x=-2$, $x=0$, $x=3$; Horizontal: $y=-2$, $y=2$

6. $\lim_{x\to0^+} f(x) = \infty$, $\lim_{x\to0^-} f(x) = -\infty$,

$\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to-\infty} f(x) = 1$



14. $\lim_{x\to5^-} \frac{e^x}{(x-5)^3} = -\infty$ since the numerator is positive and the denominator approaches 0 from the negative side as $x \to 5^-$.

18. $\lim_{x\to\infty} \frac{3x+5}{x-4} = \lim_{x\to\infty} \frac{(3x+5)/x}{(x-4)/x} = \lim_{x\to\infty} \frac{3+5/x}{1-4/x} = \frac{\lim_{x\to\infty} 3 + 5\lim_{x\to\infty}\frac{1}{x}}{\lim_{x\to\infty} 1 - 4\lim_{x\to\infty}\frac{1}{x}} = \frac{3+5(0)}{1-4(0)} = 3$

21. First, multiply the factors in the denominator. Then divide both the numerator and denominator by u^4.

$$\lim_{u\to\infty} \frac{4u^4+5}{(u^2-2)(2u^2-1)} = \lim_{u\to\infty} \frac{4u^4+5}{2u^4-5u^2+2} = \lim_{u\to\infty} \frac{\frac{4u^4+5}{u^4}}{\frac{2u^4-5u^2+2}{u^4}} = \lim_{u\to\infty} \frac{4+\frac{5}{u^4}}{2-\frac{5}{u^2}+\frac{2}{u^4}}$$

$$= \frac{\lim_{u\to\infty}\left(4+\frac{5}{u^4}\right)}{\lim_{u\to\infty}\left(2-\frac{5}{u^2}+\frac{2}{u^4}\right)} = \frac{\lim_{u\to\infty} 4 + 5\lim_{u\to\infty}\frac{1}{u^4}}{\lim_{u\to\infty} 2 - 5\lim_{u\to\infty}\frac{1}{u^2} + 2\lim_{u\to\infty}\frac{1}{u^4}} = \frac{4+5(0)}{2-5(0)+2(0)}$$

$$= \frac{4}{2} = 2$$

23. $$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right) = \lim_{x\to\infty} \frac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x} = \lim_{x\to\infty} \frac{\left(\sqrt{9x^2+x}\right)^2-(3x)^2}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty} \frac{(9x^2+x)-9x^2}{\sqrt{9x^2+x}+3x} = \lim_{x\to\infty} \frac{x/x}{\left(\sqrt{9x^2+x}+3x\right)/x} = \lim_{x\to\infty} \frac{1}{\sqrt{9+1/x}+3}$$

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

35. (a) This must be graph IV, the only graph that is always negative to the left of $x = 1$ and positive to the right of $x = 1$.

(b) This is graph III, the only graph with a horizontal asymptote of $y = 1$.

(c) This must be graph II, since it's the only graph that is positive everywhere. [$(x-1)^2 > 0$ for $x \neq 1$.]

(d) $x^2 - 1 < 0$ if $|x| < 1$. Since the function is the reciprocal of $x^2 - 1$, the graph must be negative for $|x| < 1$ and positive elsewhere. The only graph fitting this description is VI.

(e) $(x-1)^2 > 0$ if $x \neq 1$, so the sign of y is determined by the sign of the numerator, x. Thus, $y < 0$ if $x < 0$ and $y > 0$ if $x > 0$, as is only the case with graph I.

(f) The graph must have vertical asymptotes at $x = \pm 1$ and an x-intercept at $x = 0$. The only graph fitting this description is V.