4.1

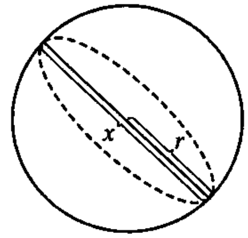
2. (a) $A = \pi r^2 \Rightarrow \dfrac{dA}{dt} = \dfrac{dA}{dr}\dfrac{dr}{dt} = 2\pi r\,\dfrac{dr}{dt}$

(b) $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} = 2\pi(30\text{ m})(1\text{ m/s}) = 60\pi\text{ m}^2\text{/s}$

5. (a) Given: the rate of decrease of the surface area is 1 cm^2/min. If we let t be time (in minutes) and S be the surface area (in cm^2), then we are given that $dS/dt = -1\text{ cm}^2/\text{s}$.

(b) Unknown: the rate of decrease of the diameter when the diameter is 10 cm. If we let x be the diameter, then we want to find dx/dt when $x = 10$ cm.

(c)



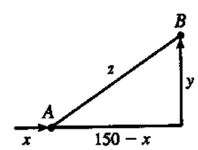
(d) If the radius is r and the diameter $x = 2r$, then $r = \frac{1}{2}x$ and $S = 4\pi r^2 = 4\pi\left(\frac{1}{2}x\right)^2 = \pi x^2 \Rightarrow$ $\dfrac{dS}{dt} = \dfrac{dS}{dx}\dfrac{dx}{dt} = 2\pi x\,\dfrac{dx}{dt}$.

(e) $-1 = \dfrac{dS}{dt} = 2\pi x\,\dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt} = -\dfrac{1}{2\pi x}$. When $x = 10$, $\dfrac{dx}{dt} = -\dfrac{1}{20\pi}$. So the rate of decrease is $\frac{1}{20\pi}$ cm/min.

6. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km/h and $dy/dt = 25$ km/h.

(b) Unknown: the rate at which the distance between the ships is changing at 4:00 P.M. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

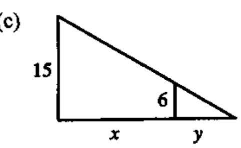
(c)



8. (a) Given: a man 6 ft tall walks away from a street light mounted on a 15-ft-tall pole at a rate of 5 ft/s. If we let t be time (in s) and x be the distance from the pole to the man (in ft), then we are given that $dx/dt = 5$ ft/s.

(b) Unknown: the rate at which the tip of his shadow is moving when he is 40 ft from the pole. If we let y be the distance from the man to the tip of his shadow (in ft), then we want to find $\dfrac{d}{dt}(x+y)$ when $x = 40$ ft.

(c)

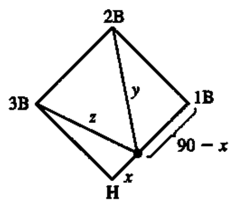


(d) By similar triangles, $\dfrac{15}{6} = \dfrac{x+y}{y} \Rightarrow 15y = 6x + 6y \Rightarrow 9y = 6x \Rightarrow y = \frac{2}{3}x$.

(e) The tip of the shadow moves at a rate of $\dfrac{d}{dt}(x+y) = \dfrac{d}{dt}\left(x + \frac{2}{3}x\right) = \dfrac{5}{3}\dfrac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/s.

12. We are given that $\frac{dx}{dt} = 24$ ft/s.

(a)



$y^2 = (90 - x)^2 + 90^2 \quad \Rightarrow 2y\,\frac{dy}{dt} = 2(90 - x)\left(-\frac{dx}{dt}\right).$

When $x = 45$, $y = \sqrt{45^2 + 90^2} = 45\sqrt{5}$, so

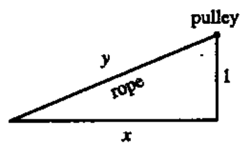
$\frac{dy}{dt} = \frac{90 - x}{y}\left(-\frac{dx}{dt}\right) = \frac{45}{45\sqrt{5}}(-24) = -\frac{24}{\sqrt{5}},$

so the distance from second base is decreasing at a rate of $\frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

(b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

$z^2 = x^2 + 90^2 \quad \Rightarrow \quad 2z\,\frac{dz}{dt} = 2x\,\frac{dx}{dt}$. When $x = 45$, $z = 45\sqrt{5}$, so $\frac{dz}{dt} = \frac{45}{45\sqrt{5}}(24) = \frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

14.



Given $\frac{dy}{dt} = -1$ m/s, find $\frac{dx}{dt}$ when $x = 8$ m. $y^2 = x^2 + 1 \quad \Rightarrow$

$2y\,\frac{dy}{dt} = 2x\,\frac{dx}{dt} \quad \Rightarrow \quad \frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt} = -\frac{y}{x}$. When $x = 8$, $y = \sqrt{65}$, so

$\frac{dx}{dt} = -\frac{\sqrt{65}}{8}$. Thus, the boat approaches the dock at $\frac{\sqrt{65}}{8} \approx 1.01$ m/s.

25. Differentiating both sides of $PV = C$ with respect to t and using the Product Rule gives us $P\,\frac{dV}{dt} + V\,\frac{dP}{dt} = 0$

$\Rightarrow \quad \frac{dV}{dt} = -\frac{V}{P}\frac{dP}{dt}$. When $V = 600$, $P = 150$ and $\frac{dP}{dt} = 20$, so we have $\frac{dV}{dt} = -\frac{600}{150}(20) = -80$. Thus, the volume is decreasing at a rate of 80 cm^3/min.