4.3

2. (a) g is concave upward on $(-1, 2)$ and $(7, 8)$.

(b) g is concave downward on $(2, 4)$ and $(4, 7)$.

(c) The only point of inflection is $(2, 2)$. Note that 7 is not in the domain of this function.

3. (a) Use the Increasing/Decreasing (I/D) Test.

(b) Use the Concavity Test.

(c) At any value of x where the concavity changes, we have an inflection point at $(x, f(x))$.

4. (a) See the First Derivative Test.

(b) See the Second Derivative Test and the note that precedes Example 5.

6. (a) f is increasing on the intervals where $f'(x) > 0$, namely, $(2, 4)$ and $(6, 9)$.

(b) f has a local maximum where it changes from increasing to decreasing, that is, where f' changes from positive to negative (at $x = 4$). Similarly, where f' changes from negative to positive, f has a local minimum (at $x = 2$ and at $x = 6$).

(c) When f' is increasing, its derivative f'' is positive and hence, f is concave upward. This happens on $(1, 3)$, $(5, 7)$, and $(8, 9)$. Similarly, f is concave downward when f' is decreasing — that is, on $(0, 1)$, $(3, 5)$, and $(7, 8)$.

(d) f has inflection points at $x = 1, 3, 5, 7$, and 8, since the direction of concavity changes at each of these values.

7. (a) $f(x) = x^3 - 12x + 1 \Rightarrow f'(x) = 3x^2 - 12 = 3(x + 2)(x - 2)$. So $f'(x) > 0 \Leftrightarrow x > 2$ or $x < -2$ and $f'(x) < 0 \Leftrightarrow -2 < x < 2$. So f is increasing on $(-\infty, -2)$ and $(2, \infty)$ and decreasing on $(-2, 2)$.

(b) f changes from increasing to decreasing at $x = -2$ and from decreasing to increasing at $x = 2$. Thus, $f(-2) = 17$ is a local maximum and $f(2) = -15$ is a local minimum.

(c) $f''(x) = 6x$. $f''(x) > 0 \Leftrightarrow x > 0$ and $f''(x) < 0 \Leftrightarrow x < 0$. Thus, f is concave upward on $(0, \infty)$ and concave downward on $(-\infty, 0)$. There is an inflection point where the concavity changes, at $(0, f(0)) = (0, 1)$.

12. (a) $y = f(x) = x^2e^x \Rightarrow f'(x) = x^2e^x + 2xe^x = x(x + 2)e^x$. So $f'(x) > 0 \Leftrightarrow x(x + 2) > 0 \Leftrightarrow$ either $x < -2$ or $x > 0$. Therefore f is increasing on $(-\infty, -2)$ and $(0, \infty)$, and decreasing on $(-2, 0)$.

(b) f changes from increasing to decreasing at $x = -2$, so $f(-2) = 4e^{-2}$ is a local maximum. f changes from decreasing to increasing at $x = 0$, so $f(0) = 0$ is a local minimum.

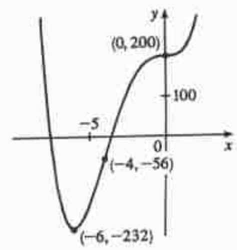
(c) $f'(x) = (x^2 + 2x)e^x \Rightarrow f''(x) = (x^2 + 2x)e^x + e^x(2x + 2) = e^x(x^2 + 4x + 2)$. $f''(x) = 0 \Leftrightarrow x^2 + 4x + 2 = 0 \Leftrightarrow x = -2 \pm \sqrt{2}$. $f''(x) < 0 \Leftrightarrow -2 - \sqrt{2} < x < -2 + \sqrt{2}$, so f is concave downward on $(-2 - \sqrt{2}, -2 + \sqrt{2})$ and concave upward on $(-\infty, -2 - \sqrt{2})$ and $(-2 + \sqrt{2}, \infty)$. There are inflection points at $(-2 - \sqrt{2}, f(-2 - \sqrt{2})) \approx (-3.41, 0.38)$ and $(-2 + \sqrt{2}, f(-2 + \sqrt{2})) \approx (-0.59, 0.19)$.

18. (a) $g(x) = 200 + 8x^3 + x^4 \quad \Rightarrow \quad g'(x) = 24x^2 + 4x^3 = 4x^2(6 + x) = 0$ when $x = -6$ and when $x = 0$. $g'(x) > 0 \quad \Leftrightarrow \quad x > -6 \; (x \neq 0)$ and $g'(x) < 0 \quad \Leftrightarrow \quad x < -6$, so g is decreasing on $(-\infty, -6)$ and g is increasing on $(-6, \infty)$, with a horizontal tangent at $x = 0$.

(b) $g(-6) = -232$ is a local minimum value. There is no local maximum value.

(c) $g''(x) = 48x + 12x^2 = 12x(4 + x) = 0$ when $x = -4$ and when $x = 0$. $g''(x) > 0 \quad \Leftrightarrow \quad x < -4$ or $x > 0$ and $g''(x) < 0 \quad \Leftrightarrow \quad -4 < x < 0$, so g is CU on $(-\infty, -4)$ and $(0, \infty)$, and g is CD on $(-4, 0)$. Inflection points at $(-4, -56)$ and $(0, 200)$

(d)



29. (a) $\lim_{x \to \pm\infty} e^{-1/(x+1)} = 1$ since $-1/(x+1) \to 0$, so $y = 1$ is a HA. $\lim_{x \to -1^+} e^{-1/(x+1)} = 0$ since $-1/(x+1) \to -\infty$, $\lim_{x \to -1^-} e^{-1/(x+1)} = \infty$ since $-1/(x+1) \to \infty$, so $x = -1$ is a VA.

(b) $f(x) = e^{-1/(x+1)} \quad \Rightarrow \quad f'(x) = e^{-1/(x+1)} \left[-(-1)\dfrac{1}{(x+1)^2} \right]$ [Reciprocal Rule] $= e^{-1/(x+1)}/(x+1)^2$

$\Rightarrow \quad f'(x) > 0$ for all x except -1, so f is increasing on $(-\infty, -1)$ and $(-1, \infty)$.

(c) No local maximum or minimum

(d) $$f''(x) = \frac{(x+1)^2 e^{-1/(x+1)}\left[1/(x+1)^2\right] - e^{-1/(x+1)}\left[2(x+1)\right]}{[(x+1)^2]^2}$$

$$= \frac{e^{-1/(x+1)}\left[1 - (2x+2)\right]}{(x+1)^4} = -\frac{e^{-1/(x+1)}(2x+1)}{(x+1)^4} \quad \Rightarrow$$

$f''(x) > 0 \quad \Leftrightarrow \quad 2x + 1 < 0 \quad \Leftrightarrow \quad x < -\frac{1}{2}$, so f is CU on $(-\infty, -1)$ and $\left(-1, -\frac{1}{2}\right)$, and CD on $\left(-\frac{1}{2}, \infty\right)$. f has an IP at $\left(-\frac{1}{2}, e^{-2}\right)$.

(e)