Name of Student: .....

(Your Instructor's Name: .....)

First Mid-Term of Math 1a

October 17, 2000 (Tuesday)
7 p.m. - 9 p.m., Science Center Hall C & E

Instructors: Yum-Tong Siu (course head), Peter Clark, Kim Froyshov
Deepee Khosla, Yang Liu, Russell Mann
David Savitt, Kiril Selverov, Yuhan Zha

Question	Points	Score
1	13	
2	13	
3	13	
4	12	
5	13	
6	12	
7	12	
8	12	
Total	100	

- *You have TWO hours to complete this examination.*
- *No calculators are allowed.*
- *No partial credit can be given for unsubstantiated answers.*
- *Use the back of the page if more space is needed for your answer (with an indication that your answer is continued on the back of the page).*

1. Let $f(x) = |x+2| - 2|x| + |x-3|$.

   (a) Evaluate $f(-5)$, $f(\frac{1}{2})$, and $f(3)$.

   (b) Sketch the graph of $f(x)$.

   (c) Let $g(x) = \frac{1}{x}$ and $h(x) = \sqrt{x-3}$. Find the natural domains of $g \circ f$ and $h \circ f$. (Hint: refer to your graph!)

2. Sketch the graph of

$$f(x) = \frac{x^2 - 1}{|x| - 1}.$$

Where is f continuous? Are there any removable discontinuities? (Recall: a point is a removable discontinuity if the function f becomes continuous after we change its value at that point.)

3. Let $f(x) = \frac{3-x}{x^2-2x-8}$. Evaluate the following limits.

(a) $\lim_{x\to\infty} f(x)$. (b) $\lim_{x\to-\infty} f(x)$. (c) $\lim_{x\to1^+} f(x)$.

(d) $\lim_{x\to1^-} f(x)$. (e) $\lim_{x\to4^+} f(x)$. (f) $\lim_{x\to4^-} f(x)$.

4. Compute the derivative of $f(x) = \frac{x^3-1}{x^2+x}$.

5. Compute $f'(x)$ and $f''(x)$ when $f(x) = \sin^2(x^4 + 1)$.

6. Find all lines tangent to the graph of $y = x^2$ which pass through the point $(1, -3)$.

7. Suppose $g(1) = 4$, $g'(1) = 3$, and $g''(1) = -2$. Suppose also that $f(4) = 6$, $f'(4) = -1$, and $f''(4) = 5$. What are the values of the first and second derivatives of $(f \circ g)(x)$ at $x = 1$?

8. Let

$$f(x) = \begin{cases} x^n \sin\left(\frac{1}{x^2}\right) & \text{for } x \neq 0 \\ \\ 0 & \text{for } x = 0 \end{cases}.$$

(a) Find the smallest integer value for n such that $f(x)$ is continuous at $x = 0$.

(b) Find the smallest integer value for n such that $f(x)$ is differentiable at $x = 0$.

(c) Find the smallest integer value for n such that $f''(x)$ exists at $x = 0$.