Name: _____

Math 1a Final Exam Thursday, January 15, 1998

Section (circle one):

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Question	Score
1	
2	
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Total	

The exam will be 3 hours long. Each question is worth ten points.

No calculators are allowed.

Justify your answers carefully. No partial credit can be given for unsubstantiated answers.

1a. Evaluate the limit of $\ln(\cos x)/x^2$ as x approaches 0.

1b. Find the derivative of the function $f(x) = \tan^{-1}(2x + 1)$.

1c. Differentiate the function $f(x) = x^{\ln(x)}$. Hint: consider ln(f).

2a. Here are four candidates for the definition of the derivative of the function f(x) at the point x=a. Circle the ones which are correct.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad \lim_{x \to a} \frac{f(x) - f(a)}{x}$$

$$\lim_{h \to 0} \frac{f(x+2h) - f(x)}{2h} \qquad \lim_{x \to a} f'(x) - f(a)$$

b. Find all lines through the origin which are tangent to the curve

$$x^2 - 4x + y^2 + 2 = 0 .$$

Note that the origin is not a point on this curve.

3. An object travels along a straight line, starting at time t=0, with position given by the formula:

$$s(t) = (2/3)t^3 - 7t^2 + 20t + 8.$$

When the velocity is positive, it is moving to the right.

a. Find a formula for the velocity at time t.

b. Find a formula for the acceleration at time t.

c. When is the object moving to the right?

d. When is the velocity decreasing?

e. When does the object change direction?

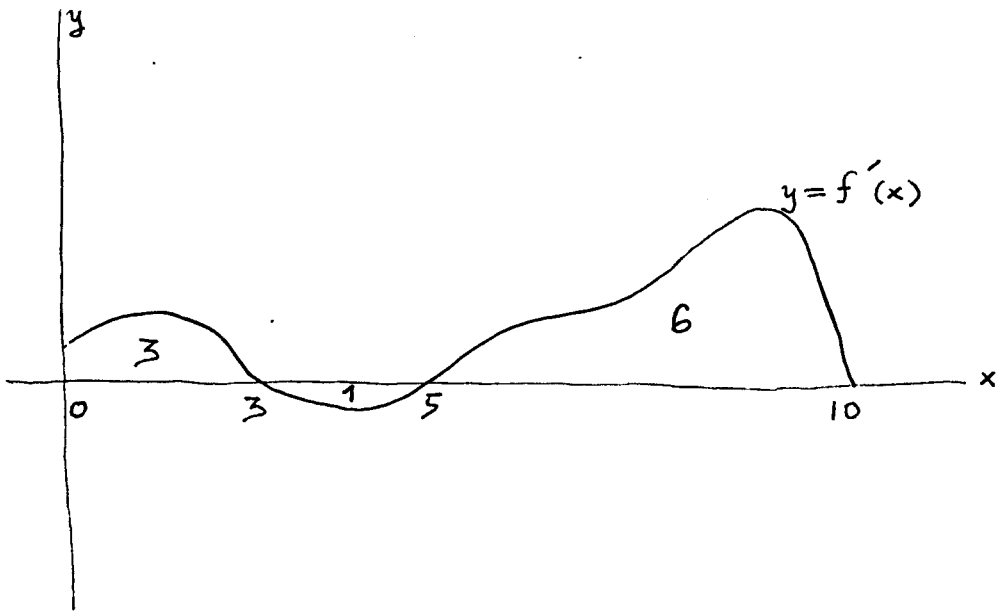
4. The derivative f'(x) of the function f(x) is graphed below. The numbers indicate the area between the graph of f'(x) and the x-axis. Assume that f(5) = 5.

a) Calculate the value f(0) =

b) Calculate the integral $\int_3^{10} f''(x)\,dx$ =

c) Where does the function f(x) take its maximum value on the interval [0,10] ?

d) What is the maximum value of f(x) on this interval?



5. I wish to enclose a rectangular area in my back yard, using 160 feet of fence. What dimensions will give the largest area in the following two situations? Make sure to verify that your answer is a maximum value.

a. The fence is on all four sides of the enclosed area.

b. The fence is on three sides of the enclosed area, and the fourth side is bounded by the back wall of my house.

6 a. Find a function f(x), defined for $x > 0$, whose derivative is equal to $1/x$ and which satisfies $f(1) = 0$.

b. For the function f(x) in part a), evaluate $f(e^{11}/e^{7})$.

c. Show how we may obtain estimates for the constant f(2) using numerical integration of the function $g(x) = 1/x$ over the interval [1,2]. Set up the sums for four equal subdivisions of this interval, using both the trapezoid rule and Simpson's rule. You do not have to evaluate the sums.

7. Let $f(x) = 3 \cdot 2^x$.

a) Find the value: $f(5) =$

b) Find the derivative of $f(x)$.

Let $g(y)$ be the inverse function to $f(x)$, defined for $y > 0$.

c) Find the values: $g(3) =$

$g(192) =$

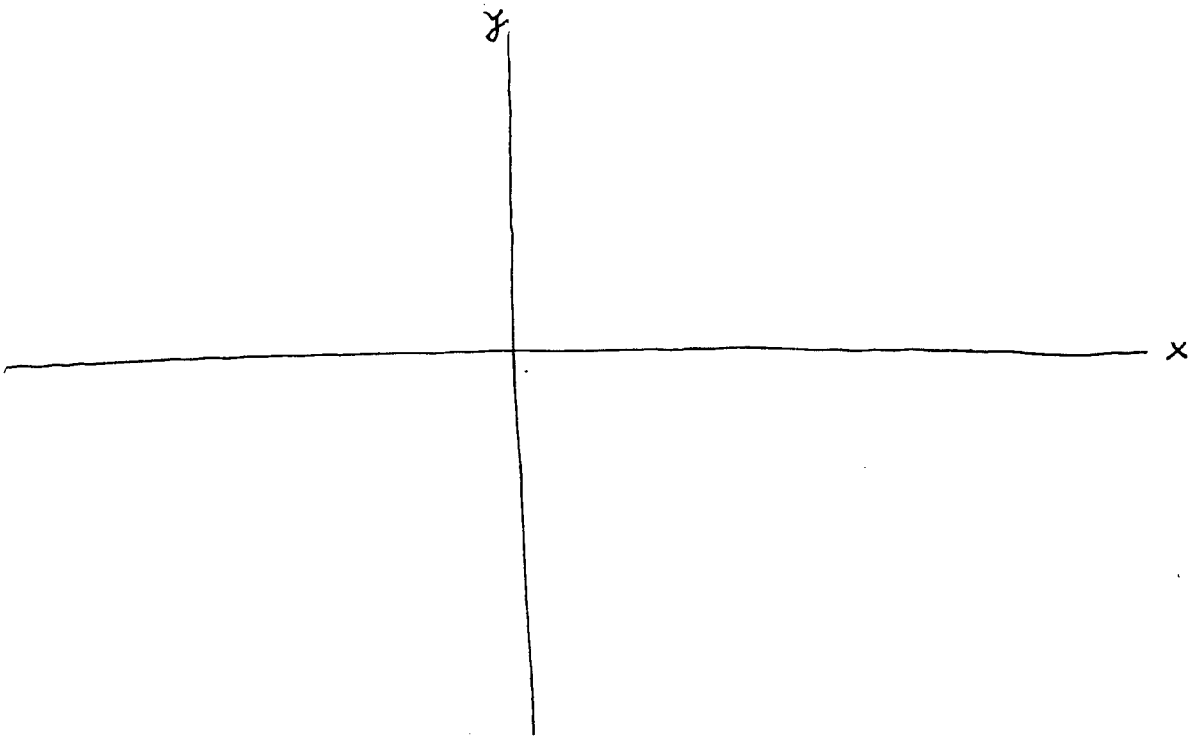
d) Find the derivative of $g(y)$, either by identifying the function $g(y)$ explicitly, or by using the chain rule.

8. Consider the function $f(x) = \int_0^x (t^2 - 3t + 2)\, dt$.

a. Find and classify the critical points of f(x).

b. What are the points of inflection?

c. Sketch the graph of f(x) below, with the x and y values of the points in a) and b) clearly marked.



9. A population of 200 toads is released in a remote area, with an unlimited food supply. Assume that the population grows exponentially according to the rule $P(t) = Ae^{kt}$, where t is the number of years after the release, and that there are 800 toads two years after the release.

a. Find a formula for the function P(t), and calculate the population 5 years after the release.

b. What is the instantaneous rate of growth of the population, five years after the release?

c. Assume that the population spreads in a circular pattern, so that the density in toads per square mile remains constant. If the radius of the initial population is 1 mile, what is the radius of the circular population of the toad colony after 5 years?

10. You are sitting in the grandstand of a racetrack, 100 feet from the track. A car races by at 220 ft/sec (about 150 mph), and you turn your head to follow the car.

How fast is your head turning (in radians per second) when the car is 240 feet down the track, a little more than one second later?