Name: _____ ID#: _____

Midterm Examination I

Math 1a
Introduction to Calculus

12 March 2004

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

> *Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.*
>
> —*Handbook for Students*

Problem Number	Possible Points	Points Earned
1	12	
2	12	
3	20	
4	21	
5	12	
6	9	
7	14	
Total	100	

1 1

1. (12 Points) Find the following:

(i) $\log_3 243$

(ii) $\log_2 \frac{1}{64}$

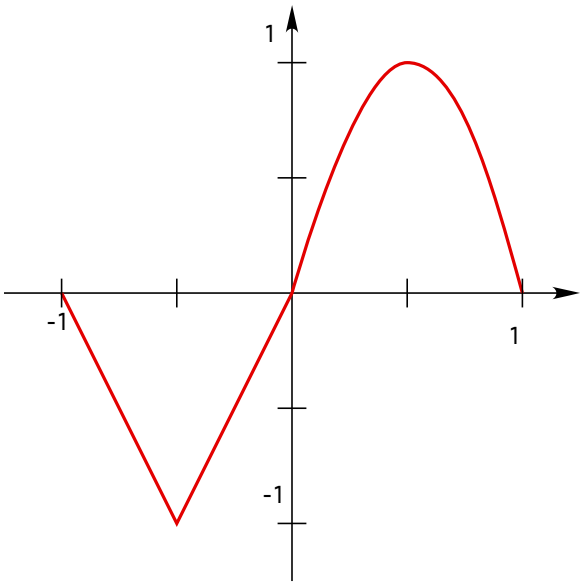
_____ / 12

2 2

2. (12 Points) Let $f(x) = \sqrt{4 - 3x^2}$. Find the domain and range of f. Explain your answers!

_____ / 12

3. (20 Points) Depicted is the graph of a function f.



Draw the graphs of the following functions, labeling the endpoints of the domain as well as the maximum and minimum points.

(i) $g(x) = f(x + 1)$

3 **3**

(ii) $h(x) = f(x) + 2$

(iii) $k(x) = f(3x)$

3

3

(iv) $\ell(x) = 4f(x)$.

(v) $m(x) = f(5x) - 6$.

_____ / 20

4 4

4. (21 Points) Define

$$f(x) = \begin{cases} x+4 & x \leq 0; \\ (x-2)^2 & 0 < x \leq 2; \\ 4 & x > 2. \end{cases}$$

(a) Find the following:

(i) $\lim_{x \to 0+} f(x)$

(ii) $\lim_{x \to 0-} f(x)$

(iii) $\lim_{x \to 0} f(x)$

4

4

(iv) $\lim_{x \to 2-} f(x)$

(v) $\lim_{x \to 2+} f(x)$

(vi) $\lim_{x \to 2} f(x)$

4 4

(b) At which points is f discontinuous? Why?

_____ / 21

5 5

5. (12 Points) Assume the limit $\lim_{h \to 0} \dfrac{2^h - 1}{h}$ exists. Call this number r.

Let $f(x) = 2^x$. Use the definition of the derivative to determine $f'(a)$ (your answer will involve r).

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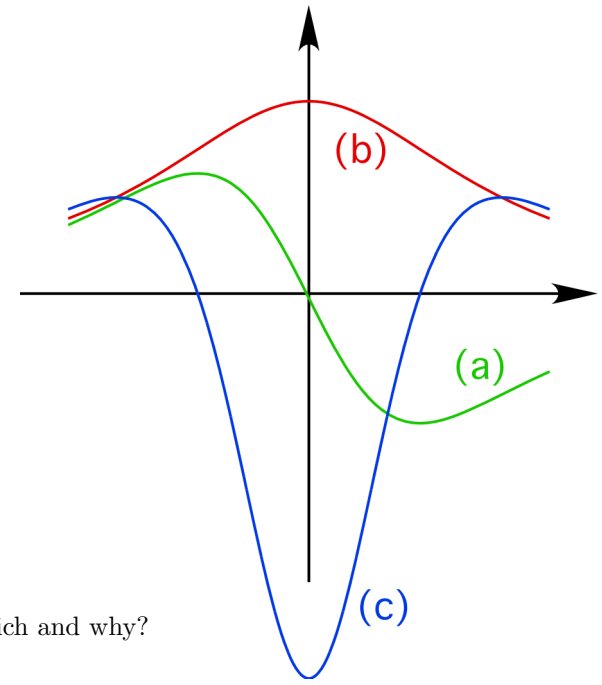
6 6

6. (9 Points) Below are graphed three functions. One of them is a function f, another is f', and the third is f''.



Which is which and why?

(a) _____

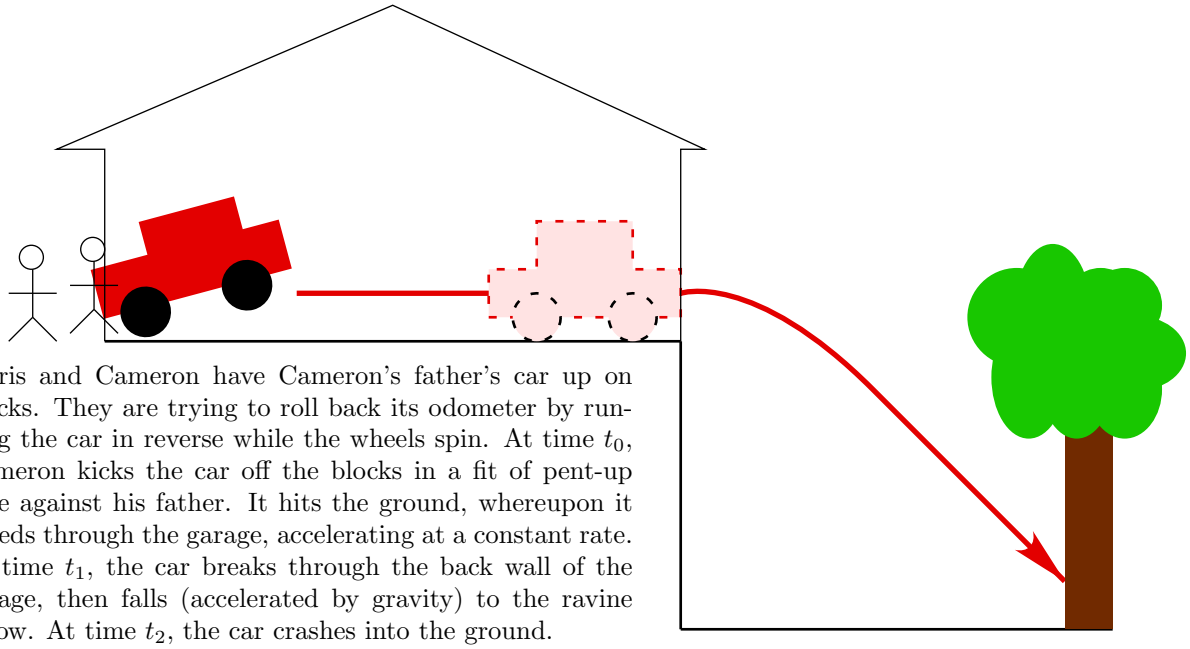
(b) _____

(c) _____

_____ / 9

7 7

7. (14 Points)



Ferris and Cameron have Cameron's father's car up on blocks. They are trying to roll back its odometer by running the car in reverse while the wheels spin. At time t_0, Cameron kicks the car off the blocks in a fit of pent-up rage against his father. It hits the ground, whereupon it speeds through the garage, accelerating at a constant rate. At time t_1, the car breaks through the back wall of the garage, then falls (accelerated by gravity) to the ravine below. At time t_2, the car crashes into the ground.

(a) Graph the acceleration of the car in the *horizontal* direction over the time interval $[t_0, t_2]$.

(b) Graph the acceleration of the car in the *vertical* direction over the same time interval.

7 7

(c) Graph the velocity of the car in the *horizontal* direction.

(d) Graph the velocity of the car in the *vertical* direction.

☐ Check the box if you know what movie this problem comes from.

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