Name of Student: .....

(Your Instructor's Name: .....)

Second Mid-Term of Math 1a

November 14, 2000 (Tuesday)
7 p.m. - 9 p.m., Science Center Hall C & E

Instructors: Yum-Tong Siu (course head), Peter Clark, Kim Froyshov
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David Savitt, Kiril Selverov, Yuhan Zha

Question	Points	Score
1	15	
2	15	
3	12	
4	20	
5	12	
6	14	
7	12	
Total	100	

- *You have TWO hours to complete this examination.*
- *No calculators are allowed.*
- *No partial credit can be given for unsubstantiated answers.*
- *Use the back of the page if more space is needed for your answer (with an indication that your answer is continued on the back of the page).*

1. Compute

   (a) $\frac{\log_5 16 \cdot \log_2 9}{\log_5 3}$,

   (b) $\arcsin(\cos(3))$,

   (c) $\cos\left(\arctan\left(\frac{5}{12}\right)\right)$.

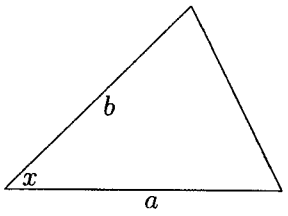
2. (a) Calculate $\frac{d}{dx}\left[\ln \arccos(x)\right]$.

   (b) Calculate $\frac{d}{dx}\left[\arctan(x)\right]^x$.

   (c) Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 0$ when x and y satisfy

$$y^3 + y\sin x - 1 = 0.$$

3. A triangle with sides a, b and angle x between them is evolving in time. The angle x is assumed to be acute (*i.e.*, less than a right angle). The area of the triangle remains fixed at $\frac{1}{2}$ square inches. Suppose the side a changes at the constant rate of 1 inch per second and the side b changes at the constant rate of 2 inches per second, what is the rate of change of x when $a = 1, b = \sqrt{2}$. (Hint: The area of the triangle is $\frac{1}{2}ab\sin x$.)



4. (a) Compute

$$\lim_{x\to 0} \frac{\sin x - x}{x^3}.$$

(b) Compute

$$\lim_{x\to\infty} (1+x)^{1/x},$$

(c) Compute

$$\lim_{x\to\infty} (1+1/x)^{x},$$

(d) Compute

$$\lim_{x\to 2} \frac{3^x - 9}{x-2}.$$

(e) Suppose $f(x)$ is a function on $(-1,1)$ and $f''(x)$ is continuous on $(-1,1)$. Suppose $f(0) = 1$, $f'(0) = 2$, and $f''(0) = 3$. Find numbers a and b such that

$$\lim_{x\to 0} \frac{5f(ax) + 7bf(x) - 12f(0)}{x^2}$$

exists and is finite. What is the limit? (Hint: Use L'Hôpital's rule.)

5. Find the intervals where the function

$$f(x) = 2x^3 - 9x^2 + 12x - 4$$

(a) is increasing

(b) is decreasing

(c) is concave up

(d) is concave down

and find the inflection points and relative extrema of $f(x)$. Sketch the graph of $f(x)$. How many solutions does $f(x)$ have?

6. (a) Find the smallest and largest values that the function

$$f(x) = x^3 - 3x + 1$$

takes on the interval $[0, 2]$.

(b) Consider the function

$$f(x) = \tan^{-1} x + \frac{4}{x+2}, \qquad x > -2.$$

(i) Find all the points x where the relative extrema of f occur.

(ii) Determine all the horizontal and vertical asymptotes of f.

(iii) Find all the points x where the absolute extrema of f occur.

7. Lara is driving due east at 15 metres per second and Lauren is driving due north at 20 metres per second. They are approaching the same intersection: in fact, when Lara is 40 metres away from the intersection, Lauren is 45 metres away from it.

   (a) If they continue at the same velocities, what will be the minimum distance between the two cars as they pass through the intersection?

   (b) How fast are they moving away from each other at that time? (*i.e.*, What is the rate of change of their distance when their distance assumes its minimum value?)