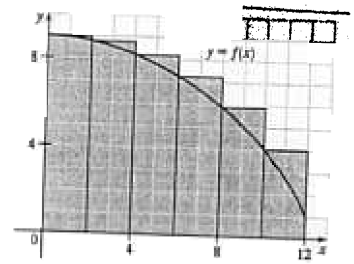
Math 1A Fall 2001: Section 5.1 Solutions

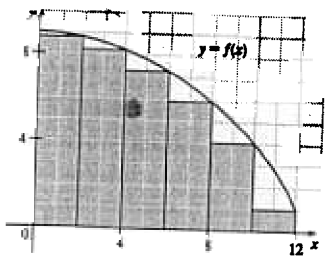
2. (a) (i) $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x \quad [\Delta x = \frac{12-0}{6} = 2]$

$$= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$$
$$= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$$
$$\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1)$$
$$= 2(43.3) = 86.6$$



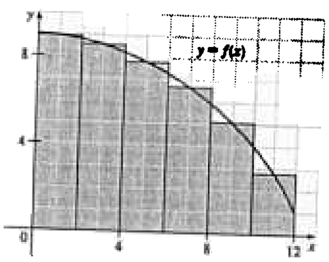
(ii) $R_6 = L_6 + 2 \cdot f(12) - 2 \cdot f(0)$

$$\approx 86.6 + 2(1) - 2(9) = 70.6$$



(iii) $M_6 = \sum_{i=1}^{6} f(x_i^*)\,\Delta x$

$$= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)]$$
$$\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8)$$
$$= 2(39.7) = 79.4$$



(b) Since f is *decreasing*, we obtain an *overestimate* by using *left* endpoints; that is, L_6.

(c) Since f is *decreasing*, we obtain an *underestimate* by using *right* endpoints; that is, R_6.

(d) M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6.

$[\Delta x = \frac{5-0}{5} = 1]$

(b) $L_5 = \sum_{i=1}^{5} f(x_{i-1})\,\Delta x$

$$= f(0) + f(1) + f(2) + f(3) + f(4)$$
$$= 25 + 24 + 21 + 16 + 9 = 95$$

L_5 is an overestimate.

12. We can find an upper estimate by using the final velocity for each time interval. Thus, the distance d traveled after 62 seconds can be approximated by

$$d = \sum_{i=1}^{6} v(t_i)\Delta t_i = (185 \text{ ft/s})(10 \text{ s}) + 319 \cdot 5 + 447 \cdot 5 + 742 \cdot 12 + 1325 \cdot 27 + 1445 \cdot 3 = 54{,}694 \text{ ft}$$

16. $f(x) = \dfrac{\ln x}{x}$, $3 \le x \le 10$. $\Delta x = (10-3)/n = 7/n$ and $x_i = 3 + i\,\Delta x = 3 + 7i/n$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\ln(3+7i/n)}{3+7i/n} \cdot \frac{7}{n}.$$

18. (a) $\Delta x = \dfrac{1-0}{n} = \dfrac{1}{n}$ and $x_i = 0 + i\,\Delta x = \dfrac{i}{n}$. $A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\dfrac{i}{n}\right)^3 \cdot \dfrac{1}{n}$.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^3} \cdot \frac{1}{n} = \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3 = \lim_{n\to\infty} \frac{1}{n^4} \left[\frac{n(n+1)}{2}\right]^2 = \lim_{n\to\infty} \frac{(n+1)^2}{4n^2} = \frac{1}{4} \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^2 = \frac{1}{4}$$