

Math 1A Fall 2001: First Math 1A Exam, Fall 1996

[1] Sketch the graph of the function

$$f(x) = x^2 + \frac{2}{x}$$

Label all intercepts, extrema, and points of inflection (give their co-ordinates), and pay attention to limits and asymptotes.

[2] Find two positive numbers whose sum is 12 and such that the product of one of the numbers with the square of the other is as large as possible.

[3] An observer on a straight beach is watching a hovercraft which is travelling parallel to the shore, two miles out to sea, at a speed of thirty miles per hour. At what rate is the observer's head turning when the hovercraft passes directly in front of the observer?

[4] (a) Find the following limit (if it exists):

$$\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos(3x)}$$

(b) The graph of a certain function  $f$  has slope  $\sin x + 7x^{12} + 4$  at every point  $(x, y)$  on the graph, and contains the point  $(0, \pi)$ . Find the function  $f$ .

[4] (c) Let  $f$  be a function such that  $f'(a)$  and  $f''(a)$  exist at a point  $a$ . Find

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{2h}$$

[5] Consider a function  $f(x)$  which is continuous on the closed interval  $[p, q]$  and differentiable on the open interval  $(p, q)$ . We are told that  $f(p) = f(q) = 0$ , and that the graph of  $f(x)$  is concave down on  $(p, q)$ . When answering the following questions you may use the Mean Value Theorem.

(a) Prove that the function  $f(x)$  has a critical point  $r$  in the interval  $(p, q)$ .

(b) What is the sign of  $f'(x)$  if  $x$  is less than  $r$ ? What if  $x$  is greater than  $r$ ?

[5] (c) What can you say about the sign of  $f(x)$  for  $x$  in  $(p, q)$ ?