Math 1A Fall 2001: Section 5.3 Solutions

2. $\int_a^b I(t)\,dt = \int_a^b Q'(t)\,dt = Q(b) - Q(a)$ by the Total Change Theorem, so it represents the change in the charge Q from time $t = a$ to $t = b$.

6. The slope of the trail is the rate of change of the elevation E, so $f(x) = E'(x)$. By the Total Change Theorem, $\int_3^5 f(x)\,dx = \int_3^5 E'(x)\,dx = E(5) - E(3)$ is the change in the elevation E between $x = 3$ miles and $x = 5$ miles from the start of the trail.

14. $\int_\pi^{2\pi} \cos\theta\,d\theta = [\sin\theta]_\pi^{2\pi} = \sin 2\pi - \sin\pi = 0 - 0 = 0$

16. $\displaystyle\int_0^1 x^{3/7}\,dx = \left[\frac{x^{10/7}}{10/7}\right]_0^1 = \left[\tfrac{7}{10}x^{10/7}\right]_0^1 = \frac{7}{10} - 0 = \frac{7}{10}$

20. $\int_0^2 (x^3-1)^2\,dx = \int_0^2 (x^6 - 2x^3 + 1)\,dx = \left[\tfrac{1}{7}x^7 - 2\left(\tfrac{1}{4}x^4\right) + x\right]_0^2 = \left(\tfrac{128}{7} - 2\cdot 4 + 2\right) - 0 = \tfrac{86}{7}$

28. $\int_{\ln 3}^{\ln 6} 8e^x\,dx = [8e^x]_{\ln 3}^{\ln 6} = 8\left(e^{\ln 6} - e^{\ln 3}\right) = 8(6-3) = 24$

34.
$$\begin{aligned}\int_{-1}^2 |x - x^2|\,dx &= \int_{-1}^0 (x^2 - x)\,dx + \int_0^1 (x - x^2)\,dx + \int_1^2 (x^2 - x)\,dx \\ &= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{-1}^0 + \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2 \\ &= 0 - \left(-\tfrac{1}{3} - \tfrac{1}{2}\right) + \left(\tfrac{1}{2} - \tfrac{1}{3}\right) - 0 + \left(\tfrac{8}{3} - 2\right) - \left(\tfrac{1}{3} - \tfrac{1}{2}\right) = \tfrac{7}{3} + \tfrac{3}{2} - 2 = \tfrac{11}{6}\end{aligned}$$

46. $\int x(1 + 2x^4)\,dx = \int (x + 2x^5)\,dx = \dfrac{x^2}{2} + 2\dfrac{x^6}{6} + C = \tfrac{1}{2}x^2 + \tfrac{1}{3}x^6 + C$

60. Let w be the amount of water in the tank. We are given that the rate of water leaving the tank is $r(t) = -dw/dt$. So by the Total Change Theorem, the total loss of water from the tank after four hours is $w(0) - w(4) = -[w(4) - w(0)] = -\int_0^4 w'(t)\,dt = \int_0^4 r(t)\,dt$. We use the Midpoint Rule with $n = 4$ and $\Delta t = 1$: $\int_0^4 r(t)\,dt \approx \sum_{i=1}^4 r(\bar{t}_i)(1) = r(0.5) + r(1.5) + r(2.5) + r(3.5) \approx 5.9 + 5.4 + 4.7 + 3.6 = 19.6$ L.