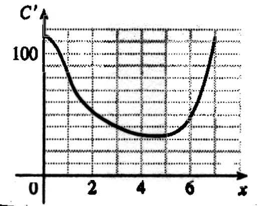
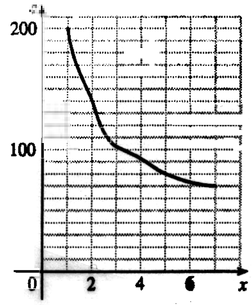
Math 1A Fall 2001: Section 4.7 Solutions

2. (a) We graph C' as in Example 1 in Section 2.8.



(b) By reading values of $C(x)$ from its graph, we can plot $c(x) = C(x)/x$.



(c) Since the graph in part (b) is decreasing, we estimate that the minimum value of $c(x)$ occurs at $x = 7$. The average cost and the marginal cost are equal at that value. See the box preceding Example 1.

6. (a) $C(x) = 2\sqrt{x} + \dfrac{x^2}{8000}$, $C(1000) = \$188.25$. $c(x) = \dfrac{C(x)}{x} = \dfrac{2}{\sqrt{x}} + \dfrac{x}{8000}$, $c(1000) = \$0.19$/unit.

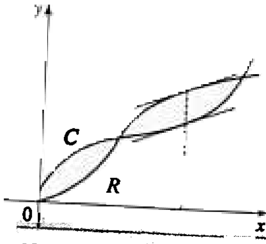
$C'(x) = \dfrac{1}{\sqrt{x}} + \dfrac{x}{4000}$, $C'(1000) = \$0.28$/unit.

(b) We must have $C'(x) = c(x) \;\Leftrightarrow\; \dfrac{1}{\sqrt{x}} + \dfrac{x}{4000} = \dfrac{2}{\sqrt{x}} + \dfrac{x}{8000} \;\Leftrightarrow\; \dfrac{x}{8000} = \dfrac{1}{\sqrt{x}} \;\Rightarrow$

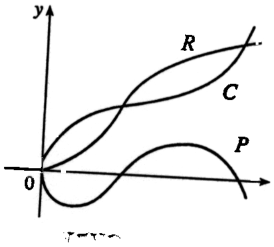
$x^{3/2} = 8000 \;\Rightarrow\; x = 8000^{2/3} = 400$. This is a minimum since $c''(x) = \frac{3}{2}x^{-5/2} > 0$.

(c) The minimum average cost is $c(400) = \$0.15$/unit.

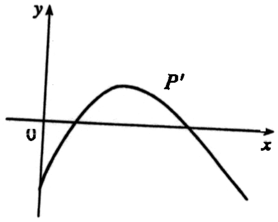
4. (a) Profit is maximized when the marginal revenue is equal to the marginal cost; that is, when R and C have equal slopes. See the box preceding Example 2.



(b) $P(x) = R(x) - C(x)$ is sketched.



(c) The marginal profit function is defined as $P'(x)$.



10. $C(x) = 16{,}000 + 500x - 1.6x^2 + 0.004x^3$, $p(x) = 1700 - 7x$. Then $R(x) = xp(x) = 1700x - 7x^2$. If the profit is maximum, then $R'(x) = C'(x) \;\Leftrightarrow\; 1700 - 14x = 500 - 3.2x + 0.012x^2 \;\Leftrightarrow$ $0.012x^2 + 10.8x - 1200 = 0 \;\Leftrightarrow\; x^2 + 900x - 100{,}000 = 0 \;\Leftrightarrow\; (x + 1000)(x - 100) = 0 \;\Leftrightarrow\; x = 100$ (since $x > 0$). The profit is maximized if $P''(x) < 0$, but since $P''(x) = R''(x) - C''(x)$, we can just check the condition $R''(x) < C''(x)$. Now $R''(x) = -14 < -3.2 + 0.024x = C''(x)$ for $x > 0$, so there is a maximum at $x = 100$.

12. $C(x) = 0.0002x^3 - 0.25x^2 + 4x + 1500$. The marginal cost is $C'(x) = 0.0006x^2 - 0.50x + 4$. $C'(x)$ is increasing when $C''(x) > 0 \;\Leftrightarrow\; 0.0012x - 0.5 > 0 \;\Leftrightarrow\; x > 0.5/0.0012 \approx 417$. So $C'(x)$ starts to increase when $x = 417$.

14. (a) Cost = setup cost + manufacturing cost $\Rightarrow$ $C(x) = 500 + m(x) = 500 + 20x - 5x^{3/4} + 0.01x^2$. We can solve $x(p) = 320 - 7.7p$ for p in terms of x to find the demand (or price) function.

$x = 320 - 7.7p \;\Rightarrow\; 7.7p = 320 - x \;\Rightarrow\; p(x) = \dfrac{320 - x}{7.7}$. $R(x) = xp(x) = \dfrac{320x - x^2}{7.7}$

(b) $C'(x) = R'(x) \;\Rightarrow\; 20 - \frac{15}{4}x^{-1/4} + 0.02x = \dfrac{320 - 2x}{7.7} \;\Rightarrow\; x \approx 81.53$ planes, and $p(x) = \$30.97$ million. The maximum profit associated with these values is about \$463.59 million.

18. Let x denote the number of \$10 increases in rent. Then the price is $p(x) = 800 + 10x$, and the number of units occupied is $100 - x$. Now the revenue is

$$R(x) = (\text{rental price per unit}) \times (\text{number of units rented})$$

$$= (800 + 10x)(100 - x) = -10x^2 + 200x + 80{,}000 \text{ for } 0 \le x \le 100 \;\Rightarrow$$

$R'(x) = -20x + 200 = 0 \;\Leftrightarrow\; x = 10$. This is a maximum since $R''(x) = -20 < 0$ for all x. Now we must check the value of $R(x) = (800 + 10x)(100 - x)$ at $x = 10$ and at the endpoints of the domain to see which value of x gives the maximum value of R. $R(0) = 80{,}000$, $R(10) = (900)(90) = 81{,}000$, and $R(100) = (1800)(0) = 0$. Thus, the maximum revenue of \$81,000/week occurs when 90 units are occupied at a