HW # 19 Solutions

$\boxed{6.2}$ 46) To minimize total cost we want to minimize the cost per mile.

Let $E = \frac{C}{V} = \frac{1}{V}(a + bV^n) = \frac{a}{V} + bV^{n-1}$. Notice that E is in dollars/mile.

$$\frac{dE}{dt} = -\frac{a}{V^2} + (n-1)bV^{n-2} = 0 \longrightarrow \frac{a}{V^2} = (n-1)bV^{n-2}$$

$$a = (n-1)bV^n \longrightarrow V^n = \frac{a}{(n-1)b} \longrightarrow \boxed{V = \left(\frac{a}{(n-1)b}\right)^{\frac{1}{n}}}$$

Compare this critical point to the endpoints:

$$\left.\begin{array}{l}\text{At a speed } V=0,\ E \to \infty \\ \text{As } V \to \infty,\ E \to \infty\end{array}\right\} \text{So the critical pt. must be a min.}$$

52) Let s be the distance from the point (x,y) to $(0,9)$.

$$s^2 = (x-0)^2 + (y-9)^2 \qquad x = 2y^2$$

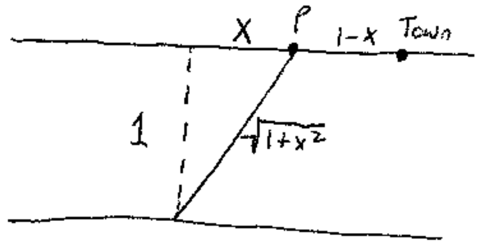
$$s^2 = (2y^2)^2 + y^2 - 18y + 81 = 4y^4 + y^2 - 18y + 81$$

$$\frac{ds^2}{dy} = 16y^3 + 2y - 18 = 0$$

$$(y-1)(16y^2 + 16y + 18) = 0 \qquad y = 1 \qquad x = 2 \qquad \boxed{(2,1)}$$

We know this is a min b/c $s^2 \to \infty$ as $x \to \pm\infty$

56) Let a be the rowing speed and b be the walking speed.



$$\text{time} = T = \left(\frac{1}{a}\right)\sqrt{1+x^2} + \left(\frac{1}{b}\right)(1-x)$$

$$\frac{dT}{dx} = \left(\frac{1}{a}\right)\left(\frac{1}{2\sqrt{1+x^2}}\right)(2x) + \left(\frac{1}{b}\right)(-1)$$

$$= \frac{x}{a\sqrt{1+x^2}} - \frac{1}{b} = 0 \longrightarrow \frac{x}{a\sqrt{1+x^2}} = \frac{1}{b} \longrightarrow bx = a\sqrt{1+x^2}$$

$$b^2x^2 = a^2(1+x^2) \longrightarrow b^2x^2 - a^2x^2 = a^2 \longrightarrow x = \sqrt{\frac{a^2}{(b^2-a^2)}}$$

Part a) $x = \sqrt{\frac{(3)^2}{(5)^2-(3)^2}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Part b) $x = \sqrt{\frac{4^2}{5^2-4^2}} = \sqrt{\frac{16}{9}} = \frac{4}{3} \longrightarrow$ Row straight to town.